AFIT/GCA/LAS/99S-2

THE DEVELOPMENT OF LASER COST ESTIMATING RELATIONSHIPS (CERs) FROM COMMERCIAL DATA

THESIS

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THESIS

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Steven L. Seeley, Capt, USAF

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Abstract

The Post-Cold War acquisition environment has been marked by significant budget reductions and the greater use of commercial practices. In this austere and rapidly changing acquisition environment, accurate cost estimates are paramount to maximize the use of the DoD's critical resources.

The advancement of laser technology has spawned a large variety of military laser applications such as the Airborne Laser (ABL), Space-Based Laser (SBL) and a host of other smaller programs. Furthermore, the Air Force Laboratory's Directed Energy Applications for Tactical Airborne Combat (DE ATAC) Study [the sponsoring activity of this thesis research] is considering a host of airborne laser system proposals. While the need for laser cost models/data is clearly increasing, such information is not readily available. Commercial pricing data is used in this research to create models that help to fill this void.

Specifically, this thesis built a commercial laser database containing commercial laser parameters and prices gathered from commercial manufacturer catalogs. An evaluation of the data identified some laser parameters as cost drivers. Using the technique of Ordinary Least Squares Regression, a series of six CERs were developed to estimate laser device prices.

THE DEVELOPMENT OF LASER COST ESTIMATING RELATIONSHIPS (CERs) FROM COMMERCIAL DATA

I. Introduction

General Problem and Background

"Technological superiority is the backbone of our military advantage," said Sheila E. Widnall, former Secretary of the Air Force. During the Cold War, the United States faced an enemy that boasted overwhelming numerical superiority. To counter those strengths, the U.S. chose to use advanced technology combined with an aggressive training program. One of these advanced military technologies is the *Laser* (1997: Year in Review, 1998:11).

For many years, the laser was touted as a "solution in search of a problem," as most of the early applications occurred in the research laboratory. However, over the past twenty years, the laser has come of age as a technological solution to many commercial as well as military problems. Commercially, laser technology has advanced a variety of areas such as medicine, telecommunications, industrial welding and cutting, and data processing. Everyday, we all come in contact with lasers in such commonplace forms as laser bar code scanners, compact disc players, and laser pointers (Rogers, 1997:7).

The military has also been quick to utilize lasers in a variety of forms. In October 1997, the U.S. Air Force got the go-ahead to house and to test a high-energy laser. The

Airborne Laser Program (ABL) is an $11 billion effort to develop, within a decade, a fleet of Boeing 747s capable of intercepting ballistic missiles by lasing them from hundreds of miles away (Donnelly, 1999). In the early 21st century, the U.S. Air Force plans on deploying the airborne attack laser. The ABL will circle over friendly airspace at 40,000 feet and destroy multiple tactical ballistic missiles shortly after they are launched hundreds of miles away (1997: Year in Review, 1998:11). The ABL represents only one of many promising, upcoming military laser applications. Other possible DoD laser applications include using lasers to clear space debris, satellite traffic management, and laser rocket propulsion (Rogers, 1997:27).

In June 1998, more than 200 representatives from the Air Force and industry attended the kickoff of the Directed Energy Applications for Tactical Airborne Combat Study (DE ATAC). The purpose of the study, chaired by retired Air Force Chief of Staff General Ronald R. Fogleman, is to look at new airborne uses for directed energy weapons (AFRL, 1998:1).

The DE ATAC study has two primary objectives. First, the study will attempt to identify promising ways in which directed energies, such as lasers, can be used from airborne platforms in tactical roles. Secondly, the study will attempt to map the path the Air Force must take in terms of technology, cost and effectiveness to develop these new weapons (AFRL, 1998:1).

According to study leader Mr. Bill Thompson, of the Directed Energy Directorate of the Air Force Research Laboratory at Kirtland Air Force Base, New Mexico, "We'll be looking exclusively at directed-energy concepts at a range of power levels, to address

weapon and mission-support applications. We'll also be considering a variety of airborne mediums, from manned aircraft to remotely piloted vehicles" (AFRL, 1998:2).

The current study is occurring in two phases. The concept definition phase (Phase I) will consider a range of potential applications, determining requirements and constraints for each concept. Directed energy technology concepts will be defined to meet the requirements and constraints of each concept application. The first phase will result in the identification of the most promising applications and technology concepts. The most promising concepts will be chosen based on technical feasibility, platform impact, mission priority, and cost-effective implementation. A concept evaluation phase (Phase II) will reconvene DE ATAC panel members to further develop and evaluate the selected concepts from Phase I (AFRL, 1998:2-3).

The final results of the DE ATAC study will identify high payoff projects for future war fighting and also produce technology development roadmaps (AFRL, 1998:3). Though the initial concept selections were made in February 1999 involving rough order of magnitude estimates, more refined models developed in this thesis research will assist in final selections.

The genesis for this thesis effort came from the cost panel portion of the DE ATAC study. The DE ATAC cost panel's purpose was to establish the following with respect to cost estimating: 1) approach, 2) structure, 3) ground rules, 4) required system definition, and 5) documentation requirements. The final objective of the cost panel is to review the cost estimates and ensure the criteria are consistent, realistic, and complete (DE ATAC Study, 1998:24). The DE ATAC applications are only conceptual and there are few military laser weapon systems to use as an analogy or to build an engineering

estimate. Therefore, this thesis develops a parametric cost estimating relationship using commercial laser price data obtained from various commercial manufacturers.

The End of the Cold War and the Advance of Laser Technology

As in many defense issues, the demise of the Cold War brought dramatic changes to United States laser funding and strategy. No longer were large defense budgets the norm. During this period of shrinking budgets, commercial and military laser technology has experienced dramatic advances. The combination of the Cold War's demise, smaller defense budgets, and advancing laser technology is reformulating the laser's military role.

One of the first and largest directed energy laser applications was the STAR WARS Program. STAR WARS, the nickname for Ronald Reagan's defensive anti-ballistic missile system, died along with the Warsaw Pact and the Cold War (Lasers, 1995:79). In a speech on March 23, 1983, President Reagan described his vision of "intercepting and destroying strategic ballistic missiles before they reached our own soil…an effort, which holds the promise of changing the course of human history" (Begley and Glick, 1992:56). The defense shield, STAR WARS, would replace the terror of mutually assured destruction with the promise of destroying any incoming missiles. However, from the beginning many physicists and military officers warned that STAR WARS was technologically impossible. The end of the Cold War meant an end to the Strategic Defense Initiative (SDI). Over $30 billion later, STAR WARS, which included laser weaponry, did not live up to its initial promise (Begley and Glick, 1992:56-57).

However, STAR WARS laid the foundation for a new horizon of laser weaponry spurred on by advances in laser technology and an infusion of new research funding (Begley and Glick, 1992:57). Today, as with STAR WARS, the missile threat is again the impetus behind the development of the laser as a defense solution. Many U.S. adversaries are increasing their long and short-range missile stockpiles. Most experts think that in the future, the enemy will possess cruise missiles like those that have worked so well for the U.S. military. Unlike the STAR WARS defense shield concept, future potential enemy missile strikes will be answered by a wide-range of U.S. laser weapons. The future laser arsenal will include planes equipped with lasers to intercept short-range missiles (Farmer and Vizard, 1997:68-69).

Military Laser Applications

In the past twenty years, lasers have solved many military problems. Indeed, the military was one of the first organizations to recognize the laser's potential (Rogers, 1997:7). The Air Force Research Laboratory and its predecessor organizations have supported laser research and development since the late 1960s. The total Air Force lab investment in laser technology over about 30 years is in excess of $1 billion (AFRL, 1998:3).

In addition to the Airborne Laser (ABL), the laser has been employed on other weapons systems such as using a laser beam to target and guide a bomb with precision. One type of precision guided-munitions (PGM), the Precision Avionics Vectoring Equipment (PAVE) series of laser target designators (LTD) and the associated PAVEWAY laser-guided bombs (LGB) have been tremendously useful in conflicts

ranging from the Vietnam War to the conflict in Kosovo. Other laser developments have been useful as training aids such as the Modulated Integrated Laser Engagement System (MILES), the military equivalent to *laser tag*. More recently, lasers have provided visible and infrared illumination for use in night vision devices (NVD) (Rogers, 1997:7).

Future military systems will most likely continue to include lasers. Laser technology and space operations have matured rapidly in the past decade offering the possibility of using lasers from space-based platforms to improve U.S. military capabilities. Laser light, used in the space environment, offers a number of unique advantages, permitting speed-of-light applications such as optical communication, illumination, target designation, active remote sensing, and high-energy weapons. Laser technology benefits more than just the military (Rogers, 1997:4).

Commercial Laser Applications

In H. G. Wells' *The War of the Worlds (1898)*, extraterrestrial aliens try to destroy Earth with their *heat ray*. This so-called heat ray was so hot and powerful that it destroys anything it touches. In 1960, when the real laser arrived, the media leaped upon its destructive power and substituted *laser* for *ray gun* to describe this new machine. The ray gun has been the popular image of a laser, and although some lasers are used as weapons or to cut metal, there are many other everyday purposes of the laser (Hecht and Teresi, 1998:1).

Most commercial laser beams can't cut or burn, but instead are just thin lines of light in the air. Simply put, it is best to think of a laser as simply a tool that uses light instead of mechanical energy. This tool's energy is powerful enough to ignite a

thermonuclear reaction or to simply drill a hole in a baby-bottle nipple (Hecht and Teresi, 1998:1-3).

The commercial laser market is very large (Table 1). The market for lasers and related equipment hit $1 billion for the first time in 1980. In addition, the 1980 sales figure doubled the 1977 sales figure (Hecht and Teresi, 1998:8). In 1998, the commercial laser industry posted global sales of $3.9 billion and is projected to increase that figure by 20% to $4.6 billion in 1999 (Anderson, 1999:80).

Commercial laser manufacturers sell lasers made from tiny semiconductor chips similar to those used in electronic circuits. The other end of the laser spectrum boasts large industrial lasers used for cutting and forming metal. The tasks that lasers perform range from the difficult to the routine; however, these tasks have a common element: they are difficult to emulate with any other tool. Commercial lasers are bought to perform a job because they deliver the necessary amount and type of energy on a particular spot. While the laser is very versatile, it can be very expensive (Hecht and Teresi, 1998:5).

Relevancy

According to General Ronald Fogleman, AF Chief of Staff, "The reality is that in the first quarter of the 21st century it will become possible to find, fix, or track and target anything that moves on the surface of the earth." Whoever has this capability will have the upper hand in any military operation, and lasers may play an important part in achieving this end (Rogers, 1997:4).

Currently, DoD is developing two major laser weapons, the Airborne Laser (ABL) and the Space-Based Laser (SBL) intended to destroy enemy ballistic missiles.

Table 1. Worldwide Commercial Nondiode-Laser Sales 1998-1999 (units) (Anderson,1999:82)

Laser Type	Year	Material Processing	Medical Therapeutic	Instrumentation	Research	Telecommunications	Optical Storage	Entertainment	Image Recording	Inspection, Measurement, & Control	Barcode Scanning	Sensing	Other	Total
CO_2 sealed	1998	5,975	3,020	0	50	0	0	0	0	0	0	250	0	9,295
	1999	6,950	2,850	0	50	0	0	0	0	0	0	250	0	10,100
CO_2 flowing	1998	3,115	0	0	10	0	0	0	0	0	0	0	0	3,125
	1999	3,311	0	0	8	0	0	0	0	0	0	0	0	3,319
Solid-State (lamp pumped)	1998	4,397	6,951	0	680	0	1	0	0	8	0	216	7	12,260
	1999	4,697	8,055	0	690	0	0	0	0	12	0	242	10	13,706
Solid-State (laser-pumped)	1998	20	0	0	394	0	0	0	0	0	0	30	0	444
	1999	40	0	0	376	0	0	0	0	0	0	30	0	446
Solid-State (diode-pumped)	1998	1,475	805	818	401	175	44	53	740	267	0	105	50	4,933
	1999	1,930	1,160	898	457	175	50	130	830	308	0	136	50	6,124
Ion < 1 W	1998	1,167	500	5,395	160	0	315	120	4,285	0	0	0	0	11,942
	1999	1,095	450	5,582	175	0	315	120	4,185	0	0	0	0	11,922
Ion > 1 W	1998	266	950	129	427	0	50	269	0	0	0	64	0	2,155
	1999	227	750	131	356	0	50	301	0	0	0	65	0	1,880
HeCd	1998	1,500	0	75	100	0	75	0	0	35	0	0	0	1,785
	1999	1,500	0	95	120	0	75	0	0	35	0	0	0	1,825
HeNe	1998	3,775	0	34,875	8,250	0	0	3,000	21,000	24,250	30,250	0	0	125,400
	1999	3,375	0	33,750	8,750	0	0	3,500	21,000	24,000	25,250	0	0	119,625
Metal Vapor	1998	20	7	0	1	0	0	0	0	20	0	0	0	48
	1999	20	7	0	1	0	0	0	0	20	0	0	0	48
Dye	1998	0	250	0	64	0	0	0	0	0	0	0	0	314
	1999	0	200	0	59	0	0	0	0	0	0	0	0	259
Excimer	1998	780	750	0	150	0	0	0	0	0	0	0	0	1,680
	1999	785	800	0	170	0	0	0	0	0	0	0	0	1,755
Total Units	1998	22,490	13,233	41,292	10,687	175	485	3,442	26,025	24,580	30,250	665	57	173,381
	1999	23,930	14,272	40,456	11,212	175	490	4,051	26,015	24,375	25,250	723	60	171,009

In addition, in a joint effort with Israel, DoD is developing a ground-based laser weapon, the Tactical High Energy Laser (THEL), that Israel will use to defend its northern cities against short-range rockets. There are also many smaller laser programs in various stages of acquisition (GAO, 1999:3). The following is a brief status of the major USAF laser efforts.

The ABL program is in the program definition and risk reduction (PDRR) acquisition phase. The seven ABL platforms are scheduled for full operational capability in 2009. The Air Force estimates the life-cycle cost of the ABL to be about $11 billion, including $3.6 billion for production (GAO, 1999:4).

The SBL program is a year into a $30-million study phase to define design concepts for the development, and deployment of a proof of concept demonstrator. The DoD estimates that it will cost $3 billion to develop and deploy the demonstrator (GAO, 1999:4).

The $131.5-million THEL Advanced Concept Technology Demonstration program is 34 months into a planned 38-month program. System components are built, but system testing was delayed from Dec 1998 to July 1999 due to administrative and technical problems. The USAF is putting $106.8-million into the program and Israel is contributing $24.7-million (GAO, 1999:4).

As researchers unveil new military laser applications and as overall defense budgets remain tight, a basic question remains - how will the costs of producing these new and sometimes leading edge systems be estimated? We do know that these new lasers will have high production risks, and hence high costs. With such large development expenditures, cost estimators will have to rely on some cost estimating method to develop reliable budgetary estimates.

Cost Estimating Basics

Cost estimates are a product of an estimating procedure that specifies the expected dollar cost to perform a stipulated task or to acquire an item system (AFMC, 1995:205).

Cost estimating is the process used to project the expected dollar cost of an item or a system (AFMC, 1995:205). One of the tenant objectives of U.S. Government acquisition is to acquire a system at a fair and reasonable price. As the DoD continues to develop and to search for new laser uses, it is and will continue to be challenged with estimating the production costs of these new systems. With few analogous systems and a lack of data to perform a grassroots estimate, the use of parametric cost estimating will play an important role in estimating the production costs of lasers. This research effort will concentrate on developing cost estimating relationships (CERs) from commercial laser price data. The mathematical models developed are a useful predictor of future laser production costs.

In March 1999, DoD Acquisition Lightening Bolt 99-3, *Market Analysis and Pricing Centers of Expertise,* called for the creation of multi-functional centers of expertise. The function of these centers will be to gather, organize, analyze, and maintain information on market products, practices, technologies, standards, and companies. This information will support the definition of requirements, the development of acquisition strategies, and the execution of price-based acquisition (PBA) (Defense Acquisition Deskbook, 1998).

What is the concept of Priced-Based Acquisition (PBA)? How does it differ from traditional cost-based acquisition? In a memorandum by Dr. Kaminski entitled, *Acquisition Reform: Mandate for Change*, he defines PBA as "a transition from a cost-based system (primarily focused on justifying cost, not reducing them) to a price or value-based system (price based on value to the customer – whatever the market will bear) to the maximum extent possible" (Defense Acquisition Deskbook, 1998).

This thesis research will apply the concept of price-based estimation by developing regression models using a commercial laser price database, to estimate the production prices of proposed military laser systems.

Research Objectives

The research objectives of this thesis are based on the question, *How do you develop CERs to estimate the production costs of a new military technology such as a laser?* While cost data may be available for the ABL and MIRACL, there doesn't appear to be any data on other laser weapon systems or laser programs. Therefore, an alternative approach was used to develop CERs based on commercial product prices and performance parameters. In order to answer this question, the authors developed a database of commercial laser catalogue data and developed CERs from that database. More specifically, the research objectives are:

A. Build a commercial laser database containing commercial laser parameters and prices gathered from commercial manufacturer catalogues.

B. Evaluate commercial laser parameters to identify laser cost drivers.

C. Perform an ordinary least squares (multivariate least squares) regression and diagnostic analysis to develop suitable explanatory models based on the commercial laser database.

Thesis Overview

This chapter provides the background of the research effort. A brief history of lasers is included. In addition, there is a discussion pertaining to military and commercial laser applications. The motivation behind this research stems from the DE ATAC study,

specifically the cost panel's search for estimating techniques for the costs of new laser applications.

Chapter II provides an in-depth review of laser cost estimating research. There is also a more thorough discussion of the history of lasers, how they work, and their military and commercial applications. In addition, commercial practice initiatives and price-based acquisition are discussed.

Chapter III contains the method used to collect and analyze the data and to run the SAS™ regression software program to develop actual cost estimating relationships. Also, included in this section is a discussion of the statistical techniques used and diagnostic techniques analyzed to determine the robustness of the developed model. Finally, there is a review of cost estimating methods and the ordinary least-squares regression technique.

Chapter IV discusses the method of data collection and laser causation. Six models were developed and analyzed using the method of Ordinary Least Squares Regression. A discussion and basic statistical output is provided following each model. Finally, detailed diagnostics were performed on our selected model.

Chapter V provides our conclusions and recommendations, based on our findings in Chapter IV, in comparison to the stated research objectives. The assumptions and relevant range of our model is also discussed. Lastly, we identified our recommendations for future research on this topic.

II. Literature Review

Overview

The literature did not contain any specific cost estimating research directly associated with lasers. Therefore, this literature review contains pertinent information about various relevant research subject areas to aid in the understanding and development of laser cost drivers. The review begins with a brief history of lasers starting with Einstein's original theories on electron stimulation. The physics of lasers is then discussed to include brief coverage of the electromagnetic spectrum, classical, and quantum physics. A discussion on how lasers actually work is included with an illustrated example of ruby laser operation. Following this discussion, the various types of lasers are reviewed from high-powered chemical lasers to lower powered semiconductor lasers. Next, commercial and military laser applications are synopsized to include such areas as the medical applications to military applications.

This literature review relies heavily on the work of Mr. Jeff Hecht for explaining the history, physical properties, and current applications of laser technology. Lastly, DoD commercial practice initiatives and price-based acquisition initiatives are discussed.

Brief History of Lasers

The laser epic starts in 1916, when the legendary physicist Albert Einstein was studying processes involving electrons. In the normal environment, electrons can either absorb or emit light. Einstein predicted that electrons could be stimulated to emit light in a particular wavelength. In 1928, Professor Ladenberg verified Einstein's stimulation

theory. However, it was not until the early 1950's that anyone seriously pursued building a practical laser device (Hecht and Teresi, 1998:50).

Although Einstein discovered what is now called *stimulated emission*, a laser also needs amplification of the stimulated emission. In the early 1950's, there were several proposals to amplify emission including those of several Soviet scientists. However, it was an American scientist named Charles H. Townes who realized what conditions would be needed to amplify the stimulated emission of microwaves. Although microwaves are in a different area of the electromagnetic spectrum than light waves, this revelation was critical to the laser's discovery (Hecht and Teresi, 1998:50).

Townes presented his idea as a thesis topic and gave it to one of his graduate students at Columbia University, James Gordon. In three years, Gordon, Townes, and Herbert Zeiger had developed the first *maser* (Microwave Amplification by Stimulated Emission of Radiation). However, the maser had few uses except for making good signal amplifiers used by radio astronomers or for satellite communications. Since microwaves weren't as practical as researchers had hoped, they began to search other areas of the electromagnetic spectrum. Researchers then became especially interested in light at infrared and visible wavelengths (Hecht and Teresi, 1998:52).

In September 1957, Townes drew a design for an "optical maser" that would emit visible light. In the summer of 1958, Townes and Arthur Schawlow applied for patents and submitted a detailed paper to the prestigious journal *Physical Review* that published it in late 1958. However, even though Townes and Schawlow had applied for patents and published a paper, no one had actually built a laser (Hecht and Teresi, 1998:52-54).

At this point, a man named Theodore Maiman, a physicist at Hughes Aircraft Company's Research Laboratories in Malibu, California, actually developed the first successful laser. Maiman built a small device in which a ruby crystal cylinder about 0.4 inches in diameter was surrounded by a helical flash lamp. The ruby rods' ends were coated to act as mirrors, which is a necessity for laser amplification (Hecht and Teresi, 1998:55).

On July 7, 1960, intense flashes of light lasting only a few millionths of a second illuminated the crystal. Maiman's device had lased by producing short pulses of light. Maiman's laser produces about 10,000 watts of light for a few millionths of a second at a time. The light produced was in the far-red end of the visible spectrum and barely visible. Nevertheless, Maiman had made history by producing the first laser (Hecht and Teresi, 1998:55-56).

When Maiman's work was published in the journal *Nature*, researchers around the world raced to build even newer types of lasers. By 1965, scientists had observed laser activity in a thousand different wavelengths in just gas media. In 1964, Townes and two Russian scientists were awarded the Nobel Prize for physics for developing the laser (Hecht and Teresi, 1998:56).

The Physics of Lasers

The word *laser* stands for Light Amplification by Stimulated Emission of Radiation (Hecht and Teresi, 1998:50). The mechanics behind laser operation is a synthesis of classical physics, quantum physics, and electromagnetic radiation theory (Hecht and Teresi, 1998:10).

Classical Physics and the Electromagnetic Spectrum

In the early 1700s, Isaac Newton discovered how light behaves. Using a rainbow as an example, he believed airborne water droplets acted as prisms by dividing the white light into the colors of the spectrum. Newton recognized that the raindrop (prism) bends some colors more than others do, so color emerges at a slightly different angle. Newton also proposed that light travels in a straight line and is made up of tiny particles (Hecht and Teresi, 1998:10).

However, Newton was only partially right. In the 1800s, Charles Huygens established that while light does travel in a straight line, it is not composed of tiny balls, but propagated in *waves*. Each wave has a different *wavelength* measured from one crest to another. In addition, each wave has a different height or *amplitude*, measured from crest to trough. Another light parameter, measured by the number of waves that pass a point during a period of time, is called the frequency (Hecht and Teresi, 1998:11).

Another key discovery about light came in the mid-1800s, when James Maxwell, whom many considered the Einstein of his day, deduced that electromagnetic forces travel in waves, as does light. Therefore, each portion of the electromagnetic spectrum must have a different wavelength. He correctly proposed that light waves are just electromagnetic waves that you can see, but there must be other portions of the spectrum that you can't see. Today, we know that the electromagnetic spectrum consists of many different types of radiation (Hecht and Teresi, 1998:11-12).

Quantum Physics

Classical physics is only an approximation of laser operational theory. To completely comprehend the inner workings of the laser, we need to look inside the atom and go into quantum physics (Hecht and Teresi, 1998:14).

In the late 1800s, a German physicist named Max Planck theorized that energy is not distributed evenly, but comes in chunks called *quanta*. Radiation is energy and so, therefore, is light. Planck then showed that light travels in waves of precise particles of energy, or photons. Whereas Newton believed light was a tiny particle, Planck showed that it is actually a blob of pure, massless, electromagnetic energy. A photon is *massless* because it travels at the speed of light and as the theory of relativity states, no particle with mass can travel so fast (Hecht and Teresi, 1998:15).

Quantum theory was a new way of analyzing matter and energy. An individual atom contains a nucleus with electrons orbiting it. These electrons move only in precise orbits around the nucleus. While electrons can move from orbit to orbit, they must move exactly into that orbit. Each orbit has a fixed energy level and the energy level of the atom depends on the orbits in which its electrons are found. Thus, there is a unique quantum number associated with each orbit, which along with the energy level, increases with the distance from the nucleus (Hecht and Teresi, 1998:15).

Why are the electrons' orbits important? The innermost orbit is said to be the ground state or lowest energy level. In order to make a *quantum* leap into a higher orbit an electron needs energy. The process of moving between orbits is called transition. If a photon has the right amount of (quantum) energy, it can move the electron into another orbit. The electron then absorbs the photon and jumps into a higher orbit. Now, the

electron and the atom are in an excited state. However, the electron can't remain in this state for long and soon drops into its ground state or lowest energy state. Upon going to the ground state, it gets rid of its extra energy by emitting a photon. The emitted photon is of the same energy and wavelength it had previously absorbed (Hecht and Teresi, 1998:16).

How Do Lasers Work?

In order for a laser to function, three things are required: 1) an active medium; 2) a population inversion; and 3) optical feedback for resonation. An active medium is a collection of atoms, molecules, or ions that emit radiation in the desired part of the spectrum. Some examples of mediums are gas or crystals (Lesson 13, 1999).

The second condition is that you must create a population inversion. A population inversion exists when more atoms are in an upper energy level than in a lower one. The active medium absorbs energy from a pumping source. This pumping source emits energy in the frequencies the active medium happens to absorb. The pump could be an arc lamp or a flash lamp. The pumping source creates stimulated absorption that excites the atom to a higher energy state. As more atoms are elevated to the higher state than are decaying from it, you get a population inversion from the normal state of the atom (Lesson 13, 1999).

As mentioned, the atoms do not stay in the high-energy excited state very long. The atoms release some of the energy in the form of a photon to get back to their ground state. There are two ways of getting back to the ground state. First, the atom may use spontaneous emission and emit a photon naturally or the atom may release the excess

energy by stimulated emission whereby a photon of the right frequency will interact with the atom and cause emission of another photon of the same frequency and phase. Therefore, stimulated emission is the process by which lasers multiply their power (Lesson 13, 1999).

The third condition for laser operation is the need for optical feedback typically called the laser resonator, which is usually a mirror. In the resonator, the laser beam is reflected back and forth increasing the stimulated emission intensity and propagating the particular wave shape of the laser, while letting a small amount escape as a laser beam (Lesson 13, 1999).

Laser Operation: An Illustrated Example

Perhaps the best way to depict how a laser works is through an illustrative example, as depicted in Figure 1, using the very first laser developed by Theodore Maiman. He started out with a rod of synthetic ruby about 1.5 inches long. One end of the rod was fully coated with silver, while the other end was partially coated with silver allowing the laser beam to escape. There was also a cooling apparatus and a power source as depicted in Figure 1. Maiman put a small spiral flash lamp around the ruby rod. When the lamp was fired, the chromium atoms in the ruby absorbed the green and blue light. This excited the majority of the chromium atoms creating a *population inversion* (Hecht and Teresi, 1998:20).

A population inversion in a ruby laser has three energy levels: the ground state, and two excited states. The flash lamp energizes the atoms to the higher of the two excited states. These atoms at the higher states quickly decay to the lower exited state.

In the lower state, the atoms remain for a fraction of a second. In the lower state, the chromium atoms in the ruby are still excited and produce stimulated emission when hit by photons of the right wavelength (Hecht and Teresi, 1998:20-22).

As previously mentioned, a few of the atoms drop down into the ground state by themselves. When these atoms do drop down they release a photon at exactly the proper wavelength. Each time one of these photons hits an excited chromium atom in the lower state, another photon of identical wavelength is released. Thus, there now are two free photons that go on to strike two more atoms beginning a chain reaction (Hecht and Teresi, 1998:22).

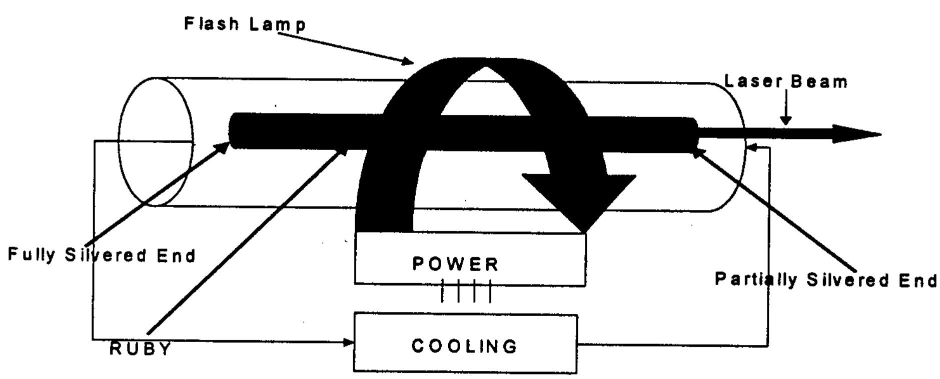


Figure 1. Basic Ruby Laser System (Hecht and Teresi, 1998:20)

Types of Lasers

There are many different types of lasers. Crystals, solids, liquids, gases and other materials are used as active media for building lasers. Several parameters describe lasers,

but wavelength is of particular importance. Each type of laser, or more generally each laser medium, emits a unique wavelength or range of wavelengths (Hecht and Teresi, 1998:30-31). For the sake of simplicity, we organized the following discussion of the types of lasers by the medium used to produce the light rather than by their wavelength.

Crystal and Glass Lasers. Maiman's ruby laser is still used today and is very typical of the crystal and glass class of lasers. Synthetic ruby is aluminum oxide containing a small amount of chromium impurity. The chromium, which gives a ruby its red color, is the active material in the laser. Ruby lasers were first used in material drilling, such as drilling holes in diamond dies. The ruby laser is still used today, but is restricted in usage by its limited power and the cooling time needed between pulses (Hecht and Teresi, 1998:31-33).

The most common crystal laser used today is the YAG laser, short for yttrium-aluminum-garnet. The YAG is made primarily of the elements yttrium, aluminum, and oxygen, and a small amount of neodymium as the active material in the crystal. Commonly, YAG lasers are used for drilling holes in metal and as military range finders (Hecht and Teresi, 1998:33).

The aforementioned are the most common crystal lasers. Other crystal lasers have been developed in the laboratory but not produced commercially. These lasers are all pumped by light from a flash lamp or another intense light source (Hecht and Teresi, 1998:34).

Gas Lasers. Today, over 5,000 gas lasers are known. Gas lasers make up such a large category that they are further divided into three subgroups by the way they are pumped (Hecht and Teresi, 1998:34).

The most common pumping method is to pass an electrical current through the gas in a tube. The method puts two electrodes on either side of the tube and applies a large voltage to get current to flow between the electrodes. Typically, a few thousand volts are used. The result, which occurs in the gas itself, is called an electrical discharge. As the electrons flow through the gas, they transfer some of their energy to the atoms in the gas, which produces a population inversion (Hecht and Teresi, 1998:35).

However, the most familiar gas laser is the helium-neon laser found in supermarket checkout lines, on construction sites to align walls, or in neon lights. This laser emits light continuously for many thousands of hours at low power (Hecht and Teresi, 1998:35).

This laser group also includes inert gas lasers such as argon and krypton. Argon lasers, the most powerful, are typically used in industry and research. Krypton lasers have a vast array of colors and are usually used in laser light shows (Hecht and Teresi, 1998:36).

The carbon-dioxide laser is more powerful than previously discussed lasers. The carbon dioxide laser has many military uses and industrial welding uses because of its high power capability. The carbon dioxide laser is unique because it uses vibrational transition rather than electrical transition. The atoms vibrate to excite the carbon dioxide in contrast to raising the electron to a higher orbit (Hecht and Teresi, 1998:36-38).

<u>Chemical Lasers</u>. A second category of gas laser includes those that are pumped by chemical reaction. One such chemical laser is the hydrogen-fluoride laser. This laser combines an atom of hydrogen with an atom of fluorine. The chemical reaction of these

two atoms produces high-power energy and an excited hydrogen-fluoride atom (Hecht and Teresi, 1998:40).

The chemical laser aboard the Airborne Laser (ABL) is called a Chemical Oxygen Iodine Laser (COIL), housed in the plane's cargo area. The laser operates by mixing hydrogen peroxide with chlorine gas to produce an excited form of oxygen called singlet delta oxygen (SDO). Iodine is then injected into the SDO to make an excited version. As the excited version relaxes to its ground energy state, a photon is produced. The photons are harvested from the lasing cavity by an optical resonator, which strips the energy from the excited iodine atoms. After the resonator, the photons are funneled through the beam-shaping optics to the beam control system to the target (Farmer and Vizard, 1997: 68-74).

Excimer Lasers. Excimer lasers are still considered a gas laser, but are a separate category because they are an intermediate between electrically driven gas lasers and chemically driven gas lasers. In an excimer laser, electrons form a beam of energy in the laser gas. The energy causes a rare gas, such as argon, to react with a halogen, such as iodine to form an excimer. An *excimer* is a molecule that exists only in an electronically excited state. The excimer emits a photon; it breaks up into its own atoms instead of going to its (nonexistent) ground state. Excimer lasers have only recently come into production, and even now are predominantly used in research. However, since they can produce high power in the ultraviolet region, these lasers are valuable in photochemistry for breaking up molecules and marking silicon wafers (Hecht and Teresi, 1998:39-40).

Semiconductor Lasers. The semiconductor laser is a close relative to the light-emitting diode (LED). Semiconductor materials, like silicon, conduct electricity better

than insulators, but not as well as true conductors. By controlling the make-up of semiconductor material, you can control how it conducts electricity. This material control is useful in complex integrated electronic circuits (Hecht and Teresi, 1998:42-43).

LEDs and semiconductor lasers are similar both in operation and electrical make-up. An electrical current excites carriers of positive and negative charges in the semiconductor and LED. These positive and negative charges combine and neutralize each other. Photons, or energy particles, are produced in the combination process of going from higher to lower energy, commonly called stimulated emission. In a semiconductor laser, the end facets are cut to reflect light, or to act as a mirror, and the operating current is higher throughout the laser. While the semiconductor laser is very efficient, cooling can be a problem. Most semiconductor lasers are small and often produced in large quantities (Hecht and Teresi, 1998:43).

Liquid Lasers. Liquids can be active laser media also. Organic dyes are dissolved in a liquid, such as alcohol, to form a solution. The liquid dye laser is a special type of laser because of the way it transitions its electrons. In all other lasers, the laser transition is between two states at fixed energy levels. However, organic dyes have energy levels so closely spaced that it practically forms a continuum. The large number of levels is due to the complexity of the dye molecules (Hecht and Teresi, 1998:45).

Laser Applications

Medical. Lasers are used in a variety of medical applications, from surgery to bleaching out tattoos. Laser surgery is ideal for delicate operations that can't be done as well with a physical medical device. The major attraction of medical lasers is that it seals

off small blood vessels as it cuts, preventing significant bleeding (Hecht and Teresi, 1998:62-64).

Most laser surgery uses carbon-dioxide lasers that emit 50 watts of light in the infrared region. The major benefit of medical lasers is that the laser's entire beam can be focused on a spot as small as 40 micrometers. The light is strongly absorbed by the water in living cells. The combination of high intensity and high absorption vaporizes the cells in the focused areas (Hecht and Teresi, 1998:64-65).

Medical laser equipment is expensive and typically used in surgeries where there is a major advantage over conventional procedures such as lack of space or high potential for excessive bleeding. Some examples of medical laser uses are gynecological surgery; and mouth, nose, eye and throat operations (Hecht and Teresi, 1998:67).

Communication. The amount of information carried by an electromagnetic wave depends on its frequency. The higher the frequency, the more information can be transmitted in a given time interval. Light as a communication medium allows higher transmission data rates. Voice frequencies transmitted by phone are only 1,000 to 4,000 cycles per second (hertz), whereas television signals have a frequency of around 50 million-hertz. On the contrary, light wave frequencies are 800 trillion-hertz. Thus, the laser offers an excellent way to transmit this light. The combination of lasers, fiber-optic, and computer technology has revolutionized communications (Hecht and Teresi, 1998:81-82).

For example, a single fiber can carry a thousand telephone conversations. In addition, the transmission capacity of optical fibers has now surpassed 1 billion hertz in lab demonstrations. Two types of light sources are used in fiber optics. One is the

semiconductor laser, which is a crystal no bigger than a grain of salt. The other is the light emitting diode (LED) which is a semiconductor-glass laser. The two are similar except the LED doesn't contain a laser resonator and emits a broader beam (Hecht and Teresi, 1998:67, 86-88).

Military. Military laser use has been a reality for almost three decades. With the advance of the ABL, MIRACL, and space-base laser applications, the military is confident in the promise of lasers. The military's use of lasers can be broken into two distinct groups: high power and low power applications. Within each group, the military is applying a wide array of both tactical and strategic laser systems. Table 2 below illustrates various DoD laser efforts over the past 30 years. None of these efforts has yet resulted in an operational laser weapon system (GAO, 1999:14-15).

The military's present and past applications of high power lasers centers around the use of high power, directed energy weapons against various enemy targets. For instance, a spy satellite is worthless without its sensitive electronic "eyes" made up of imaging equipment. High power laser beams have the potential to damage the satellite's sensitive equipment rendering it useless (Hecht and Teresi, 1998:105).

To fully understand the military's application of high power lasers, it is useful to review the laser processes used. First, it takes a lot of power to damage metal and other hard targets. However, you have to consider not just power, but also the way laser energy is transferred and coupled from beam to target. The first part of the process is simple heating because a target will absorb some of the laser energy and reflect the remainder (Hecht and Teresi, 1998:106).

The absorbed energy will heat the target and as the temperature rises, it absorbs even more of the incident laser energy. By lasing the target hard enough and long enough, the surface will melt and the evaporated metal will create a cloud of ionized gas called plasma (Hecht and Teresi, 1998:106).

Usually the laser will continue to transfer energy to the target through plasma, gradually burning a hole through the target in a matter of seconds. However, simply heating the target doesn't make the most efficient use of a laser's power. The use of a

Table 2. Examples of DoD Laser Development Efforts (GAO, 1999:14)

Development Effort	**Purpose**	**Inception Date**
Tri-Service Laser Program	Develop carbon dioxide gas dynamic laser	1968
Navy-Advanced Research Projects Agency Chemical Laser Program	Develop high-energy chemical laser	1971
Airborne Laser Laboratory	Demonstrate the feasibility of using a high-energy laser in an airborne environment	1972
Mid-Infrared Advanced Chemical Laser	Develop and integrate a ground-based high-energy chemical laser w/ a beam control system	1977
Space-Based Laser Program	Develop space-based high-energy chemical laser weapon system	1977
Ground-Based Laser (free electron)	Develop high-energy free electron laser weapon system	1979
Ground-Based Laser (Excimer)	Develop high-energy excimer laser weapon system	1979

series of high power laser pulses rather than a continuous beam will cause mechanical and thermal damage because each pulse evaporates a small amount of target material from the surface. A series of closely spaced pulses is effective because it heats a material

while also pounding it. Also, heating a material makes it less resistant to impact, which makes the pounding more effective (Hecht and Teresi, 1998:106).

The combination of thermal and mechanical damage is important because many military targets are made of aluminum sheets. Aluminum is a difficult material to damage because over 90% of the incident light from a carbon dioxide laser is reflected off of aluminum. Nevertheless, if the skin of a target has been penetrated, it may be possible to damage critical internal components or disable them by the intense laser heat (Hecht and Teresi, 1998:107). Therefore, the target is destroyed without having to use high power to blow it up (Hecht and Teresi, 1998:106-107).

The two specific applications of high-energy lasers are in the airborne medium and space medium. The ABL is designed to operate independently to intercept tactical ballistic missiles in their vulnerable boost phase, shooting down cruise and surface-to-air missiles, protecting reconnaissance aircraft from air-to-air missiles and peering deep into enemy territory with precise optics (Farmer and Vizard, 1997: 68-74).

Boeing is the prime contractor, integrating TRW's Chemical Oxygen Iodine Laser (COIL) and the Lockheed Martin Beam Director and Infrared Search and Track (IRST) systems onboard a converted 747-400F freighter. A full power flight test is scheduled in 2000. In 2002 the ABL prototype will attempt to shoot down a tactical ballistic missile. The targeting illuminator laser locks onto and begins to track the missile. On command, the high-energy COIL laser punches a hole in the target (Farmer and Vizard, 1997: 68-74).

Then Secretary of the Air Force, Sheila Widnall, said of the ABL, "This will be the first time a nation has developed a militarily effective high-energy laser weapon."

She also added, "We believe the individual technologies are proven, but we must now integrate these technologies onto an airplane with weight and operational restrictions" (Farmer and Vizard, 1997: 68-74).

The space-medium is also being considered for high-energy laser weaponry. The Army's Mid-Infrared Advanced Chemical Laser (MIRACL) is a complex array of ground-based telescopes and mirrors operated by a computerized tracking and stabilizing system. The key component of the MIRACL laser is the Hughes-made Sealite beam director, which locks the MIRACL laser onto its target. The laser is mounted on a 5.1-inch naval gun turret with a fast 350-degree swivel motion. The MIRACL uses a deuterium fluoride chemical laser. The MIRACL has already destroyed a short-range rocket over White-Sands missile range (Farmer and Vizard, 1997: 68-74).

The military's application of lower power lasers is very diverse. Laser characteristics make them ideal for targeting and identification uses. Since laser beams travel at the speed of light, which is so much faster than any existing military hardware, the leading correction required is very small over a long distance. In addition, since the beam is steered with mirrors, a laser can be shifted quickly to another target. In addition, since light travels in a straight line and is narrow, lasers are a perfect choice for targeting (Hecht and Teresi, 1998:111).

In 1979, in a congressional hearing, a high ranking DoD spokesman on science and technology said, "aiming lasers have been the single most successful investment in the last decade" (Hecht and Teresi, 1998:103).

Lasers were first appeared in battle in 1972 with the use of the first laser-guided "smart bombs" in Vietnam. Soldiers on the ground illuminated a target with a low power

laser, typically a helium-neon. A bomb-mounted sensor detects the spot and directs the bomb toward the laser-designated target (Hecht and Teresi, 1998:125).

Laser range finders measure the distance to a target by measuring the time it takes for a laser pulse to travel there and back. There are many types of lasers based on laser range-finder applications. One derivative of laser range-finder technology is the Modulated Integrated Laser Engagement System (MILES) (Hecht and Teresi, 1998:126-129).

The MILES system is a simple, but efficient concept. Instead of firing bullets at each other, soldiers fire laser light. Each gun is equipped with a small semiconductor laser that sends a coded series of energy pulses when the gun is fired that identifies the gun-type. MILES is just an example of one low power laser application. There are numerous others such as military radar, satellite communications, and countermeasures (Hecht and Teresi, 1998:129).

Manufacturing. Most manufacturing laser applications deal with some kind of materials working. The laser is an efficient tool because it doesn't contact the material. Lasers are especially useful in drilling tiny holes, making complex patterns, or in hard to get at areas since they can lay down a precise beam of energy on a tiny spot with little effect on the surrounding area. Lasers are also useful for materials that are brittle and hard (like ceramics and diamonds) or materials that are soft and easily deformed (like rubber and plastic). However, lasers are not very good at making large holes because a large hole requires more lasers and hence more power (Hecht and Teresi, 1998:132-133).

Manufacturing lasers come in both high and low power. High power lasers are predominantly used in welding and drilling because they are used to vaporize the

material. Some high power lasers used in manufacturing are YAG, neodymium-glass, ruby, and carbon dioxide. Low power lasers are used for heat-treating and annealing, quality assurance, cloth cutting, plastic working, marking, and etching (Hecht and Teresi, 1998:134-135).

Measuring and Other Uses. Low power lasers are also used in grading, drainage, irrigation, and surveying, among many other measuring and testing applications. These lasers are typically the low power helium-neon lasers (Hecht and Teresi, 1998:154).

Other low power lasers are used in Universal Product Code (UPC) scanners, inventory control, printing plate manufacture, laser printers, compact disc (CD) players, and entertainment-based laser light shows (Hecht and Teresi, 1998:153).

Now that the various laser applications have been identified, a discussion follows on the new methods to acquire future systems—commercial practices and price-based acquisition.

Commercial Practices and Price-Based Acquisition

In a memorandum by Dr. Kaminski entitled, *Acquisition Reform: Mandate for Change*, he defines PBA as "a transition from a cost-based system (primarily focused on justifying cost, not reducing them) to a price or value-based system (price based on value to the customer – whatever the market will bear) to the maximum extent possible (Defense Acquisition Deskbook, 1998).

Reducing acquisition costs by using commercial instead of military-unique practices and technologies is an ever-increasing DoD goal. The Military Products from Commercial Lines (MPCL) is a pilot program that leverages the commercial electronics-

manufacturing base. MPCL is a four-year project designed to show that high technology military hardware can be built on commercial production lines with the same durability, functionality, and reliability, and at a significantly reduced price (Heberling and others, 1998:48).

The Air Force Research Laboratory's Manufacturing Division sponsors the MPCL. TRW Avionics Systems Division is the prime contractor of the MPCL program. In the initial production test phase, USAF F-22 Raptor and RAH-66 Comanche avionics modules were redesigned using mostly commercial off-the-shelf parts. A computer integrated manufacturing (CIM) system implemented at the TRW Automotive Electronics Plant led to minimal line interruption for the set-up and changeover between military and commercial products (Heberling and others, 1998: 48).

The MPCL team designed a process for acquiring military-unique modules as Commercial Off-the-Shelf (COTS) items relying on price analysis instead of cost analysis. The Air Force and Army program participants avoided more than 50% over the baseline military hardware versions. Also, the technology led to the commercial redesign of additional F-22 modules which resulted in recurring cost reductions (Heberling and others, 1998:49).

After demonstrating the benefits gained from producing military products from commercial lines, the team focused on the next MPCL strategy, transferring the technology to industry. The MPCL team conducted two surveys to gather the necessary data. One survey was an in-depth requirement validation survey of a small number of commercial electronic manufacturing service (EMS) firms. The other survey was a broad-based commercial impact survey of more than 1,340 EMS and printed wiring

board companies. The actual survey was modeled after a typical commercial transaction for EMS services. The MPCL team built a request for quotation package (RFQ) that included business practice requirements, model contract terms and conditions, and a representative build and test quantity of MPCL modules (Heberling and others, 1998:49-50).

The MPCL validation surveys showed that several commercial suppliers could build the redesigned military hardware at a commercial price. The tight distribution of pricing data suggested that the responding firms indicated a real measure of the transferability of the MPCL commercial redesign and streamlined business practices. In addition, the average price represented a 68% savings over the military baseline for F-22 and RAH-66 versions of these modules (Heberling and others, 1998:51-52).

In another study conducted by the Lean Aerospace Initiative (LAI) at the Massachusetts Institute of Technology, program representatives from 23 defense acquisition programs were surveyed to capture the government's overall results and lessons learned in implementing commercial practices. The LAI team defined commercial practices, per the Defense Systems Management College (DSMC) definition as, "the techniques, methods, customs, processes, rules, guides, and standards normally used by business" (Anderson and Rebentisch, 1998:17).

The surveyed programs were those that the DoD and the defense industry regarded as pioneers in incorporating commercial practices into their acquisition strategies to include all branches of the DoD and the Coast Guard. The sample data included seven aircraft programs, five ship programs, four munitions programs, and seven major acquisition programs. For each program, the team interviewed front-line

government acquisition managers about their hands-on experience implementing commercial practices (Anderson and Rebentisch, 1998:17).

Ultimately, there were eight distinct commercial practices used by the programs surveyed. These practices were: past performance, best value, commercial warranties, government/contractor cooperation and relationship, performance specifications, commercial specifications and standards, streamlined contract administration, and commercial-off-the shelf/non-developmental item (COTS/NDI). Overall, the team found commercial practices afforded strong benefits in the areas of cost, schedule, and quality. There were few reported compromises to life-cycle support and life cycle costs (Anderson and Rebentisch, 1998:17).

The program representatives surveyed confirmed that their use of commercial practices indeed gave valuable program benefits. For instance, the use of commercial practices resulted in direct program savings of almost $4 billion or 4.3% per program. For comparison purposes, a 1994 DoD-sponsored Coopers and Lybrand study titled, "The DoD Regulatory Cost Premium: A Quantitative Assessment," studied 10 government contractors, substituting best commercial practices for traditional DoD practices. The savings added up to 9% of a major acquisition program's total contract cost (Anderson and Rebentisch, 1998:19).

As commercial practices reach a full and sustained implementation, detailed cost and pricing data to which DoD is accustomed will no longer be available. This, in turn, leads to the necessity for Price-Based Acquisition discussed in the previous chapter. Future estimates will therefore become more dependent on either catalog pricing for existing systems, or parametric estimating using models developed from commercial

pricing data that relate price to key performance parameters. As the laser systems being proposed in the DE ATAC study do not yet exist, the focus of this thesis is to assist in developing such parametric models.

III. Methodology

Overview

We begin this chapter by briefly discussing the various cost estimating methodologies--to include their advantages and disadvantages. We then describe the method of least squares regression, which is used by following the model building approach outlined in *Applied Linear Regression Models* (Neter and others, 1996:328).

Cost Estimating Methodologies

Since resources are limited, a large amount time and effort in planning for future acquisition is required. The central issue in planning concerns resource allocation. The acquisition process centers on the *cost estimate.* System budgets are based on the cost estimate and future cost performance is measured against the cost estimate. In addition, cost estimating must be accurate if the Biennial Planning, Programming, and Budgeting System (BPPBS) is to be a realistic and effective decision-making tool (AFMC, 1995:201).

There are several different cost estimating methodologies. Parametric cost estimating, which this research effort will employ as a cost-estimating tool involves the development and utilization of estimating relationships between historical costs and product physical and performance characteristics. The basic idea behind parametric cost estimating is that the cost of a system is related in an approximate, but quantifiable way, to the product's physical and performance characteristics. Relevant historical data is

collected at an appropriate level and is related to areas to be estimated using mathematical and statistical techniques (AFMC, 1995:209).

There are several advantages to parametric cost estimating. Parametrics is a fast estimating method that does not require detailed inputs. Parametric estimates can also reflect the impacts of program changes and problems. In addition, parametrics alone can be used to capture a major part of the estimate, which may reduce the need for other techniques (AFMC, 1995:203).

There are several disadvantages to parametric estimating. Parametrics do not provide low-level visibility into discrete areas. In addition, parametrics require an extensive database of historical costs related to system characteristics. Lastly, portions of the actual cost estimate may not be separable from the total estimate (AFMC, 1995:203).

Another cost estimating technique is called analogous estimating. This technique uses an analogous system, one that is similar to the system being estimated, to develop a cost estimate (AFMC, 1995:210). The analogy technique is quick to prepare, easy to break into lower levels, and a good crosscheck technique. The analogy technique also lets the estimator break the estimate down into lower levels and use separate analogies for different cost elements (AFMC, 1995:204).

The major disadvantage of analogous estimating is that it requires sound engineering support and requires detailed program and technical definition on both the analogous and estimated system. The use of analogies is best early in the program when actual cost data is unavailable, but program definition is sufficient and when estimating a system that is a combination of existing systems (AFMC, 1995:204).

Grass roots cost estimating is another technique. A grass-roots estimate is also known as the industrial engineering, engineering build-up, or detailed estimate. The grass-roots technique is the most detailed, time-consuming, and costly technique. This method is usually used on systems currently in production with recent costs and a stable design. This method's basis assumption is that future costs can be estimated with accuracy based on the historical system cost. The technique uses standards derived from time and motion studies, vendor quotes, man-loading requirements, and other relevant factors. The cost build-up is based on the Work Breakdown Structure (WBS), functional areas or end-items (AFMC, 1995:210).

Other cost estimating methods include catalog pricing, expert judgement, and using estimate at completion formulas involving cost and schedule variances from cost reports to predict the cost at completion (AFMC, 1995:211). Our analysis will focus on parametric methods, particularly the Method of Least Squares Regression.

The selection of the Method of Least Squares Regression to develop our price-based estimates of lasers is based on several factors. Most notably, there are not many analogous military laser weapon systems--especially airborne applications. The small number of operational military laser systems also leads to a paucity of useful cost data that would be required for an analogous cost estimate. Likewise, a grass-root estimating technique is too time consuming and expensive. Other estimating techniques such as expert judgement are preferred for top-level, rough-order budgetary estimates only. The use of parametrics is best in this case because there is no well-defined program and little detailed data.

Model Building

The primary purpose of this research topic is to create a prediction model for the price of lasers. In other words, this is an exploratory observational study. We developed our regression model using the strategy outlined below in Figure 2 (Neter and others, 1996:328). This strategy begins the process with data collection, however, prior to this stage, we needed to know what data to collect. We needed to have some idea how lasers worked and what drove their cost (the causation system); otherwise, we may have wasted a significant amount of time collecting useless data. Some initial legwork was performed before a causation system could be established. This entailed evaluating cause and effect, identifying variables, and hypothesizing their functional relationships. The best way we saw to do this was to speak to industry experts and perform an extensive literature review. We searched for explanatory variables that were highly related to the cost of lasers, with the intent of accurately predicting the cost of future laser systems. Once the causation system was established and the independent variables were identified, we began to collect data and prepare it for analysis.

Data Collection and Preparation. This stage is one of the most critical stages in the process. Regression models can only be as good as the data they are based on. Searching for data in a young industry (as in this case) makes model development even tougher. Sometimes there just isn't a lot of data out there to find.

Once the variables have been identified and the data collected, the variables need to be analyzed. According to Neter and others, the variables need to be screened and dropped when they (1) do not add explanatory power to the model, (2) have a tendency

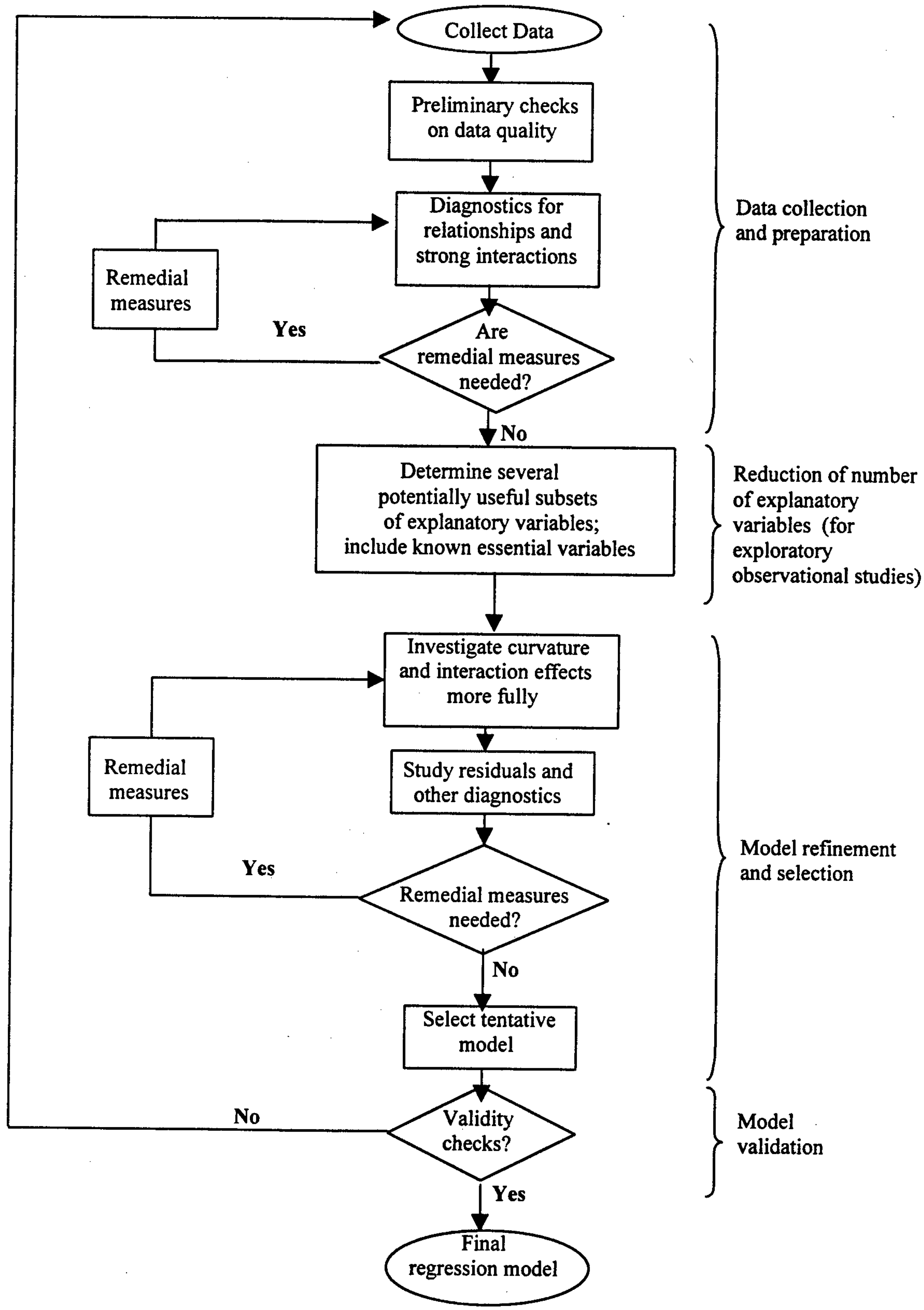


Figure 2. Strategy for Building a Regression Model

towards large measurement errors, and/or (3) duplicate another variable on the list. The data need to be checked for errors and possible outliers when plotted. If the data contains errors, every effort must be made to find the cause of the error and correct it when possible, especially when there isn't much data to begin with. If outliers are present in the scatter plots, they must also be investigated. If they cannot be explained or normalized to the rest of the data, consider dropping them. The default way to handle outliers is to leave them in the data set unless they can be dropped for a good reason. Another way to handle outliers is to weight the observations so that outlying data points carry less influence on the data set but it wasn't necessary for us to weight the data points as we had no influential outlying data points in our database.

Once the data set and variables have been established, the modeling process can begin. We recommend running a linear model first to establish a baseline for comparison purposes. Diagnostics on the results can then be performed. Some thought needs to go into the functional relationship between the predictor variables and the response variable. What is the functional form of each variable? Will the response variable increase at a constant rate with the predictor variables? Will it increase at an increasing rate? The answers to these questions will determine whether transformations of the variables need to be performed (i.e., square root, reciprocal, etc.).

Another consideration is whether there is any interaction effects between the predictor variables. Interaction occurs "when the effects of the predictor variables on the response variable are not additive, the effect of one predictor variable depends on the levels of the other predictor variables" (Neter and others, 1996:224).

Reduction of Explanatory Variables. After initial screening of the explanatory variables, there still may be too many variables to work with efficiently. Therefore, we may need to consider reducing the number of explanatory variables in our model. There are several reasons for this. A regression model with numerous explanatory variables is typically more difficult to maintain. Further, regression models with a small number of explanatory variables are easier to work with and understand. Finally, "the presence of many highly inter-correlated explanatory variables may substantially increase the sampling variation of the regression coefficients, detract from the model's descriptive abilities, increase the problem of round-off errors, and not improve, or even worsen, the model's predictive ability" (Neter and others, 1996:331).

Elimination of variables should never be taken lightly. A lot of thought and analysis needs to go into this decision. The main purpose of studying each variable is to identify the marginal explanatory power that variable adds to the model when the other variables are in the model. Numerous computerized methods statistically analyze variables to determine whether they add explanatory power to a regression model. Some of these are Forward Stepwise, Forward Selection, and Backward Elimination. The decision of which variables to include in the model should not be left up to these computerized methods alone. Other diagnostic procedures need to be considered in the refinement and selection of variables. Judgement of the analysts also plays a major part in selecting variables for an appropriate regression model.

Model Refinement and Selection. At this stage, the selected model or models need to be checked for curvature, interaction effects, multicollinearity, influential outlying observations, etc. Diagnostic tools (i.e., partial regression plots, residual

analysis, etc.) are a great aid in model refinement and selection. In fact, the selection of the best regression model depends heavily on the results of these tools. After a thorough analysis using these tools, the analysts can make a good decision on their final regression model.

Model Validation. "Model validity refers to the stability and reasonableness of the regression coefficients, the plausibility and usability of the regression function, and the ability to generalize inferences drawn from the regression analysis" (Neter and others, 1996: 334). There are three basic ways to validate a regression model: (1) collect new data to check the model for its predictive ability, (2) compare the results with expectations, and (3) use a holdout sample to check the model for its predictive ability. The best way to validate a regression model is to collect new data. We were unable to validate our model using the three methods identified above. First, we were unable to collect additional data; second, comparison with expectations is not yet realistic for DE ATAC; and finally, our database is realistically not large enough for use of a hold out sample. Therefore, for validation, this thesis relied solely on model diagnostics, which are described later.

In the process of building our model, we will use the Method of Ordinary Least Squares Regression and follow the model building strategy outlined above.

Method of Ordinary Least Squares Regression

This section will give a brief overview of the Method of Ordinary Least Squares Regression. This overview will only consider simple linear regression; however, we use

multivariate regression in our analysis. For a more detailed description of this method, please see Chapter 1 of *Applied Linear Regression Models* (Neter and others, 1996:3-35).

In performing our analysis of the cost of high power lasers, we will use the Method of Ordinary Least Squares Regression. Our goal was to determine the best estimated normal error regression model using the data we collected:

$$Y_i = B_0 + B_1 X_i + e_i \tag{1}$$

This method tries to find good estimators of the population Y-intercept parameter (B_0) and population slope parameter (B_i). Each trial will contain an X observation (x_i) and a corresponding Y observation (y_i). These are termed the predictor (independent) variable and response (dependent) variable, respectively. For each observation (x_i, y_i), this method considers the deviation of the response variable (y_i) from its expected value. In particular, we consider the sum of the n squared deviations, denoted by Q. The goal is to develop a model that is a good description of the statistical relationship between the predictor variables and the response variable.

$$Q = \sum_{i=1}^{n} (Y_i - B_0 - B_1 X_i)^2 \tag{2}$$

The fitted regression line shown in Figure 3 minimizes the squared values of the residuals for each data point. This fitted regression line may be used to predict the cost given a particular level of power.

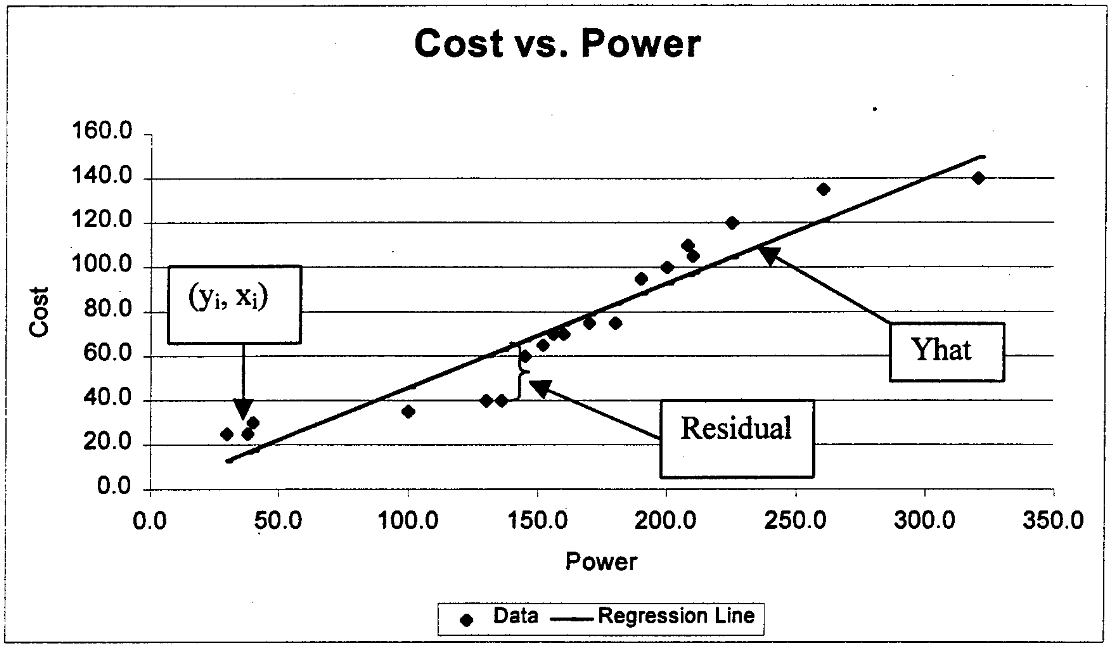


Figure 3. Sample Regression Plot

The estimators of B_0 and B_1 are b_0 and b_1, respectively. The method of least squares tries to find those values of b_0 and b_1 that minimize the sum of squared deviations (Q) for the given sample observations $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$. The estimators b_0 and b_1 can be calculated using the following equations:

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \quad \textit{where } \bar{x} \textit{ and } \bar{y} = \textit{mean value of } x\textit{'s and } y\textit{'s, respectively} \tag{3}$$

$$b_0 = \bar{y} - b_1 \bar{x} \tag{4}$$

With this information, we can now determine the predicted values (Yhat$_i$) for each trial by using the following equation:

$$\hat{Y} = b_0 + b_1 X_i \tag{5}$$

Yhat is a point estimate of the response variable (Y) when a particular level of the predictor variable (X) is entered into the equation. When all X observations are entered into the equation, there will be corresponding Yhat observations. When these points (X_i, Yhat$_i$) are plotted on a graph, the resulting line provides the least squares regression line. It is very unlikely that this line will run through every point on a scatter plot of the predictor variables and their corresponding response variables. Therefore, we will have an element of error surrounding the calculated regression line. This error is called the residual (e_i). The residual is defined as:

$$e_i = y_i - \hat{y}_i \tag{6}$$

In other words, the residual is the vertical deviation of Y_i from the fitted value Yhat$_i$ on the estimated regression line. We now have enough information to determine the estimated normal error regression model shown in Equation 1 above.

We have introduced the concept of Ordinary Least Squares Regression for a single variable; however, the actual model we are fitting has multiple variables. The basic structure and process is the same as identified as above, but the multivariate model

is slightly different. In matrix terms, the general linear regression model is (Neter and others, 1996:226):

$$\underset{n\times 1}{\mathbf{Y}} = \underset{n\times p}{\mathbf{X}} \; \underset{p\times 1}{\mathrm{B}} + \underset{n\times 1}{\varepsilon} \tag{7}$$

where:

Y is a vector of responses

B is a vector of parameters

X is a matrix of independent variables

ε is a vector of independent normal random variables with expectation $\mathbf{E}\{\varepsilon\} = 0$

and the least squares estimators are:

$$\underset{p\times 1}{\mathbf{b}} = \underset{p\times p}{(\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}} \underset{p\times 1}{(\mathbf{X}^{\mathrm{T}}\mathbf{Y})} \tag{8}$$

where:

b is an estimator of B

We will now discuss our detailed strategy using the Method of Ordinary Least Regression and the model building strategy noted above.

Detailed Strategy

As the initial step in formulating a cost estimating relationship for the price of high-power lasers, pricing data and various laser characteristics were gathered for many different lasers. This data was obtained from catalogs of various commercial laser manufacturers and consolidated in a common database. Price data was used for two

reasons: 1) because manufacturers are reluctant to provide cost data due to its proprietary nature, and 2) it is consistent with the current interest in Price Based Acquisition.

Our first challenge is to develop an adequate understanding of the causation system. In other words, what are the logical variables that drive the cost of lasers and what is their functional relationship to cost? Do they increase/decrease at a constant rate, an increasing rate, or a decreasing rate? Using the categories of data provided, we developed theories on which variables should have the biggest effect on cost in an attempt to identify the most logical cost drivers.

We then used the SAS software package to run the Least Squares Best Fit (PROC REG) procedure to determine and analyze the regression equation. Our first regression model was linear in order to establish a baseline for comparison purposes. We also used SAS to produce the partial regression plots. These plots are useful in determining whether a variable adds explanatory power to the model and provides insight into its functional relationship to the dependent variable. Categorical variables were used as required to differentiate between different types of lasers (i.e., gas, solid-state, etc.).

We also used SAS to provide standardized regression coefficients. Standardizing involves centering and scaling the variable. This will help control rounding errors, which is common when the variables have substantially different magnitudes. Standardizing the coefficients will also permit comparison of the regression coefficients in common units (standard deviations).

We analyzed our data using several statistical criteria. Below is a discussion of those we considered:

Coefficient of Multiple Determination (R^2). The coefficient of multiple determination is used to determine the amount of variation *explained* by the regression equation. The reason the word *explained* is italicized quotes is to emphasize that this word should not be taken literally. In a regression model, there is no implication that the dependent variable necessarily depends on the independent variables in a causal or explanatory sense (Neter and others, 1996:84). The R^2 is calculated by dividing the Regression Sum of Squares (SSR) by the Total Sum of Squares (SSTO). SSR is considered a measure of that part of the variability associated with the regression line. The Error Sum of Squares (SSE) is considered that part of the variability due to either random error or is explained by other variables not in the model. SSTO is the sum of SSE and SSR and includes all of the variation. R^2 values range between 0 and 1.

F-statistic (F*). F* is a measure of how strong the relation is between the dependent variable and the set of independent variables. The F* is calculated by dividing the Regression Mean Square (MSR) by the Error Mean Square (MSE). MSR is the same value as SSR. MSE is basically the SSE divided by the degrees of freedom. Large values of F* support H_a and values of F* near 1 support H_0. The test statistic used to determine whether F* is significant is:

If $F^* \leq F(1-\alpha; 1, n-2)$, conclude H_0 (The null hypothesis is that it is not significant.)

If $F^* \geq F(1-\alpha; 1, n-2)$, conclude H_a (The alternate hypothesis is that it is significant.)

Sign of Coefficients. We will look at the signs of the coefficients to ensure they make logical sense. For example, if power is included in our model and its coefficient is

negative, we know there is a problem. What this is saying is that as power increases, cost decreases. This, of course, does not make logical sense.

Variance Inflation Factor (VIF). VIF is a formal indicator of multicollinearity. A VIF value greater than 10 indicates that multicollinearity may be unduly influencing the Least Squares Best Fit estimates; therefore, the lower the VIF value the better (Neter and others, 1996:387).

t statistic (t*). The t* is used to further analyze the appropriateness of each independent variable. It helps determine the marginal effect each independent variable adds to the model. A good t* is one in which the absolute value is significantly greater than one. A quick test to determine whether the t* is significant is to look at its corresponding p-value. All tests were performed at the 95% confidence level (or 5% significance level). The p-values are generated by SAS. A p-value of 0.05 or less indicates that the 95% confidence level has been achieved.

At this point, we analyzed the data graphically (through residual plots) in order to assess the appropriateness of the model, locate any influential outliers, test for normality of the error terms, check for interaction effects, and locate any signs of non-constancy of error variance.

Appropriateness of Model. This can be determined by examining the residuals against the fitted values. If the residual plot does not suggest any systematic deviations from the response plane and the variance of the error terms appear constant (a linear, horizontal band), the model is said to be appropriate.

Outliers. Examining the residuals against the fitted values as well as the residuals against the independent variables can also identify outliers. We will discuss other

statistical tests for outliers and their influence later. For now, studying these residual plots is just a quick visual method to get an idea how many outlying points may be in our data set.

Normal Probability Plot of Residuals. This plot is a method of visualizing whether the error terms are normal. If the plot appears linear or close to linear, this suggests agreement with normality. If it is not clear that this plot is close enough to linear, a more decisive test can be conducted. This entails calculating the coefficient of multiple correlation between the ordered residuals and the expected values under normality and comparing it to the critical value in Table B-6 (Neter and others, 1996:698). This test is performed at the 95% confidence level.

Interaction Effects. To check for interaction effects, we plotted the residuals against the interaction terms among all non-qualitative variables in our earlier models. Our final model includes an interaction term; therefore, testing for interaction effects is not required. In general, if there are no systematic patterns, conclude that there are no interaction effects reflected by these terms.

Non-constancy of Error Variance. To test for non-constancy of error variance, we plotted the residuals against each of the independent variables in the model. The plots appeared in random fashion; therefore, we concluded that non-constancy of error variance does not exist. If it appeared that the error variance increases/decreases as the level of the independent variable increases/decreases, we would have conducted a more formal test.

Influence. Finally, we reviewed the data for influential outlying data points by examining several statistical measures, such as: Studentized Residuals, Studentized

Deleted Residuals, Cook's Distance, DFFITS, and DFBETAS (Neter and others, 1996:372-384). No influential outlying data points were discovered, so it was not necessary for us to research them on a case-by-case basis to determine whether they need to be corrected or dropped from the data set.

Studentized Residuals. This is an effective method for detecting outlying Y observations. It considers the magnitude of each residual relative to its standard deviation to recognize significant differences in the sampling errors of the residuals. If the studentized residual value is greater than the absolute value of 3.0, it is considered an outlying observation.

Studentized Deleted Residuals. This is a more refined method of identifying outlying Y observations. An observation is deleted from the data set so the fitted regression is based on all observations except that one. The deleted residual is then calculated, then divided by its standard deviation. This procedure is duplicated for each residual in the data set. Again, a studentized deleted residual value greater than the absolute value of 3.0 denotes an outlying observation.

Cook's Distance (Cook's D). Cook's D considers the influence each observation has on all fitted values when that observation is deleted from the data set. Each of the fitted values is compared with their corresponding fitted values when an observation is deleted. The differences are then squared and summed so the aggregate influence of the deleted observation is measured without regard to its sign. For interpreting Cook's D, it has been found useful to relate it to the F distribution and ascertain the corresponding percentile. If the percentile is near 50% or more, it implies

that the deleted observation has a major influence on the fit of the regression function (Neter and others, 1996:381).

DFFITS. This is another measure of influence, but only on the influence a deleted observation has on the fitted value for that observation. It measures the difference between the fitted value for an observation when it is in the function and its predicted value when it's omitted from the function. This difference is then divided by the error mean square of the fitted regression function when the observation is omitted. This standardizes the data. We consider our data set to be small to medium; therefore, an observation is considered influential if the absolute value of DFFITS exceeds 1.0 (Neter and others, 1996:379).

DFBETAS. This is a measure of influence of an observation on each regression coefficient. It is measured as the difference between the regression coefficient based on all observations and the regression coefficient when an observation is omitted. This difference is then divided by the standard deviation of the coefficient to obtain the DFBETA value. Again, as a guideline, a DFBETA value greater than the absolute value of 1.0 is indicative of a large impact on the regression coefficient and therefore is considered influential (Neter and others, 1996:383).

After using the Method of Ordinary Least Squares Regression, our goal is to develop a useful model to predict the cost of future laser systems given that the laser characteristics (independent variables) in our model are known. The results of our analysis are provided in Chapter IV.

IV. Analysis and Results

Examination of Data and Sources

The laser database was collected from six U.S. laser manufacturers. The database was constructed entirely of 1999 model year price and parameter information. Collection was accomplished in a variety of ways to include telephonic interviews, personal interviews, and catalogue data reviews.

During data collection, it was discovered that most of the laser manufacturers we contacted were somewhat secretive as to the product prices and parameters. This is primarily attributed to the extremely competitive environment they operate in. Consequently, many companies do not publish their product prices or all the features of each product. Therefore, if the manufacturer's catalogue did not contain the specific price or product features of a particular laser, we contacted that company's technical representatives. Some companies were cooperative while others were not. The sources of the data (i.e. corporate names) have been omitted as requested by the manufacturers to protect the proprietary nature of this information.

In order to conduct our analysis, several crucial assumptions were made. First, all data points were assumed accurate. In other words, no errors in recording or miscalculations were made in gathering the data. We also assumed that the accounting systems for collected cost data were similar for all platforms. In addition, we assumed that the causation system remained constant for all data points. Finally, a 95% confidence level or a 5% significance level was chosen as the standard for all data analysis (p-values will be reported). The models are limited by the historical data being

used to develop the model. Also, shifts in technology can effect the results being analyzed.

The following is a review and discussion of the laser database itself. Each descriptive data field and variable field is defined and briefly explained. The complete database can be found in Appendix A.

Laser Type: The commercial lasers chosen for inclusion in the database represent a variety of laser types. These include carbon dioxide (CO_2), argon ion, krypton ion, neodymium: yttritium-aluminum garnet (Nd: YAG), diode, and excimer. The inclusion of several laser types will allow the cost estimating relationship (CER) developed to be useful over a wide-array of laser applications. We identified four qualitative variables to analyze the impact of the five laser types. The variable names and associated values are given in Table 3 below.

Table 3. Qualitative Variable Values for Each Laser Type

	Qualitative Variable Values			
Laser Type	CO2	ION	YAG	DIODE
CO_2	1	0	0	0
Ion	0	1	0	0
Nd: YAG	0	0	1	0
Diode	0	0	0	1
Excimer	0	0	0	0

Price (PRICE): The prices collected were in calendar year 1999 U.S. dollars. As mentioned, the laser prices were either collected using company catalogues or by actually contacting a corporate technical representative. The prices represent the retail price of buying one laser. The number of lasers purchased is important to note because the unit price of lasers tends to drop significantly if purchased in bulk (generally in quantities greater than ten units). Each laser chosen had a production run over a similar range of units per year.

Application: The laser applications field is diverse and represents a wide-range of laser uses. These include lasers used for Original Equipment Manufacturing (OEM), cutting, drilling, holography, lightshows, marking, soldering, spot welding, stream welding, research, and industrial usage, to name a few.

Output Power (POWER): Power is the rate at which energy is produced. The unit of power is the watt or the number of joules produced per second. The output power of a laser depends on whether the output is continuous or pulsed (Laurence, 1986:31). The database consists of lasers ranging in output power from as little as 1.6 watts (e.g. OEM) to 8000 watts (e.g. welding) (Laurence, 1986:31).

Wavelength (WAVE): The term *wavelength* is used to describe electromagnetic waves. All lasers use wavelength to describe one cycle of oscillation, or the distance from one crest to another crest. Wavelengths are measured in nanometers (10^{-9} meters) or micrometers (10^{-6} meters). This variable represents the wavelengths each laser was designed for in a specific application. We converted all of the data to nanometers (Laurence, 1986:31)

Efficiency (EFFIC): The efficiency parameter is a measure of how well a laser produces its power. Limitations on efficiency are inherent in the nature of lasers. The overall efficiency of lasers is low by electrical standards, although comparable with that of other light sources. A typical commercial laser converts anywhere from 0.001% to 30% of input electrical energy into the laser beam. Meanwhile, an ordinary incandescent lamp emits only 3-4% of its electrical energy as visible light (other energy is emitted in the infrared spectrum) (Hecht, 1992:33).

Weight (WEIGHT): This parameter is the laser device's weight measured in pounds as stated by the manufacturer. A key distinction must be made between the weight of the laser itself versus the weight of the whole laser system. For instance, a medical laser device has periphery equipment that is included in the system. However, this database consists of only the weight of the laser device itself, not the whole laser application system.

Beam Divergence (BEAMDIV): The angular spreading of a laser beam with distance. A laser beam has a certain diameter and the further the beam travels, the larger that diameter becomes. For example, think of the light emanating from a flashlight. That spreading of light is called divergence. "Beam divergence typically is measured in milliradians, or thousandth of a radian. A radian equals 57.3 degrees; 2π radians equals 360 degrees, or a full circle" (Hecht, 1992:14).

Laser System Work Breakdown Structure (WBS)

The equations developed calculate the commercial price of a single laser device. Military lasers typically are not stand-alone items, but instead are part of a system.

Military laser systems are made up of several other components such as the platform (ground, ship, aircraft, and spacecraft), support equipment and other components. In addition to the component costs, there are costs related to such areas as training, data, and site activation, to name a few. There are also costs related to various phases such as concept exploration, program definition and risk reduction (PDRR), engineering and manufacturing development (EMD), production, and operation and support. So, how can you gather all of the costs necessary to build upon a commercial laser price to create a laser weapon system cost estimate?

In the literature search, there were some commercial software packages that have the capability to estimate not only hardware cost, but also the cost of the whole system. Some examples of these estimating packages are PRICE H™ and SEER H™. However, the algorithms and factors, which create these estimates, are proprietary and require licensing fees to use.

Since the commercial software packages are proprietary and the literature search revealed no other published algorithms or factors, the use of an appropriate laser work breakdown structure (WBS) would be useful as a starting point. The following represents a production WBS structure for military laser applications. The complete WBS structure is located in Appendix H (DE ATAC Study, 1998). The bold text indicates the part of the WBS we feel is captured by our CER.

WBS#	WBS Item

2.0 DE ATAC Production Phase
- 2.1 Aircraft System
  - 2.1.1 Air Vehicle
  - 2.1.2 Platform Modification
  - 2.1.3 Training
  - 2.1.4 Peculiar Support Equipment (PSE)
  - 2.1.5 System Test and Evaluation
  - 2.1.6 Systems Engineering and Project Management
  - 2.1.7 Data
  - 2.1.8 Operational Site Activation
  - 2.1.9 Common Support Equipment
  - 2.1.10 Industrial Facilities
- 2.2 Payload
  - **2.2.1 Laser Device**
  - 2.2.2 Laser Mission Equipment
  - 2.2.3 Training
  - 2.2.4 Peculiar Support Equipment (PSE)
  - 2.2.5 System Test and Evaluation
  - 2.2.6 Systems Engineering and Project Management
  - 2.2.7 Data
  - 2.2.8 Operational Site Activation
  - 2.2.9 Common Support Equipment
  - 2.2.10 Industrial Facilities
- 2.3 Payload/Platform Integration
- 2.4 Engineering Change Orders
- 2.5 SPO Support
- 2.6 Warranty

A possible framework to estimate the cost of a military laser system could be developed using this effort's equations as a foundation. If, perhaps, some analogous cost data or actual laser cost data could be found, cost factors could be developed based on this effort's models and the WBS structure previously discussed. However, this area is immature and further research is needed to develop a more detailed framework to estimate the entire laser system.

Laser Causation

The first step in our analysis was to plot each of the independent variables in our database against the dependent variable (PRICE) to identify those variables that may have a direct relationship towards price.

As can be seen from Figure 4 below, there appears to be a strong relationship between the output power of lasers and the price of those lasers. This makes intuitive sense. The more power you need, the costlier the laser. In our view, there seems to be a slight curvilinear relationship. As power increases, price seems to increase at a decreasing rate. We will investigate this variable further as it appears a likely candidate for our regression model.

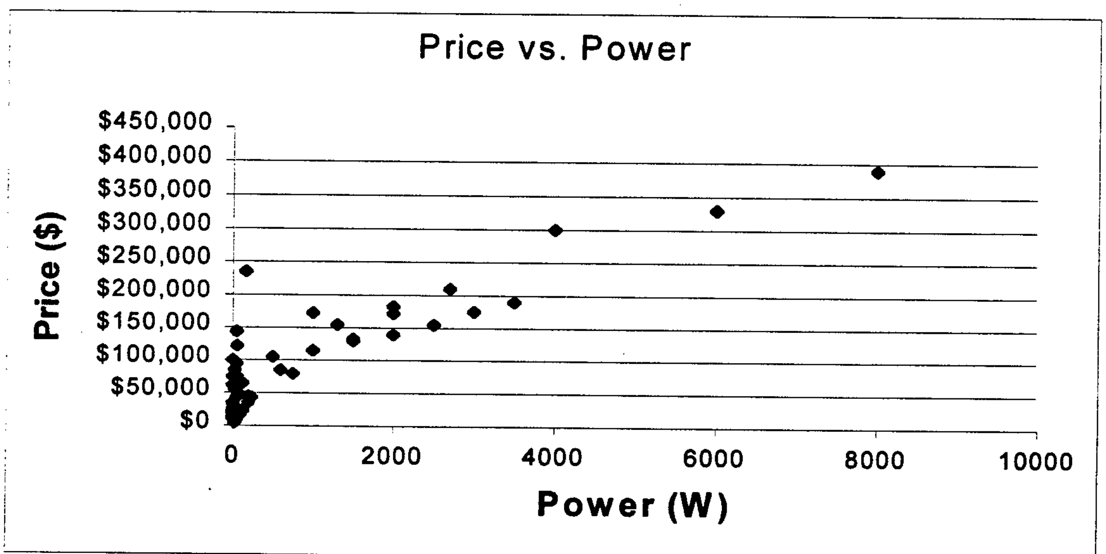


Figure 4. Scatter Plot of Price vs. Power

As can be seen from Figure 5 below, wavelength tends to depend on the type of laser being used. There appears to be no important relationship between wavelength and price. For example, the CO_2 lasers shown above are all at roughly 10,640nm and the

price varies greatly from \$5,000 to \$390,000. There are obviously other factors involved, such as power, which explains the price differential. Wavelength may be a cost driver within a particular type of laser that can operate over a range of wavelengths, such as Ion lasers, however that is beyond the scope of this research. Based on this information, we will rule out the use of wavelength in our model.

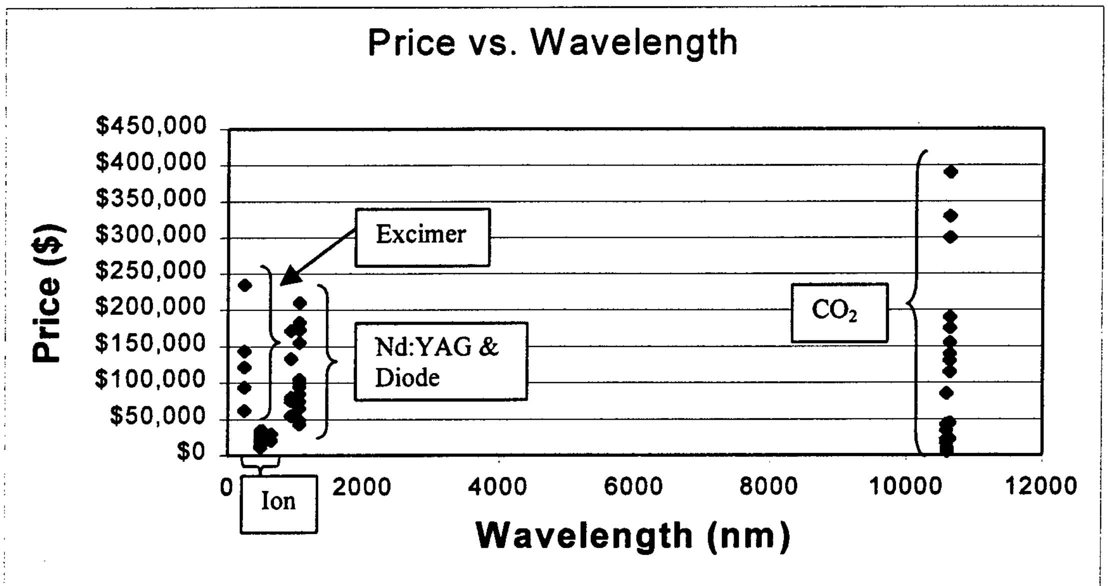


Figure 5. Scatter Plot of Price vs. Wavelength

Efficiency (power out divided by power in) appears to be another variable that depends, in part, on the type of laser being used (See Figure 6). There seems to be no clear relation between price and efficiency across all of the lasers in our database. However, in the case of CO_2 lasers, there appears to be a counter-intuitive relationship. The more efficient lasers are actually less expensive. There are obviously other factors involved that drive the price of lasers. Based on this data, we will no longer consider using efficiency as a variable in our model.

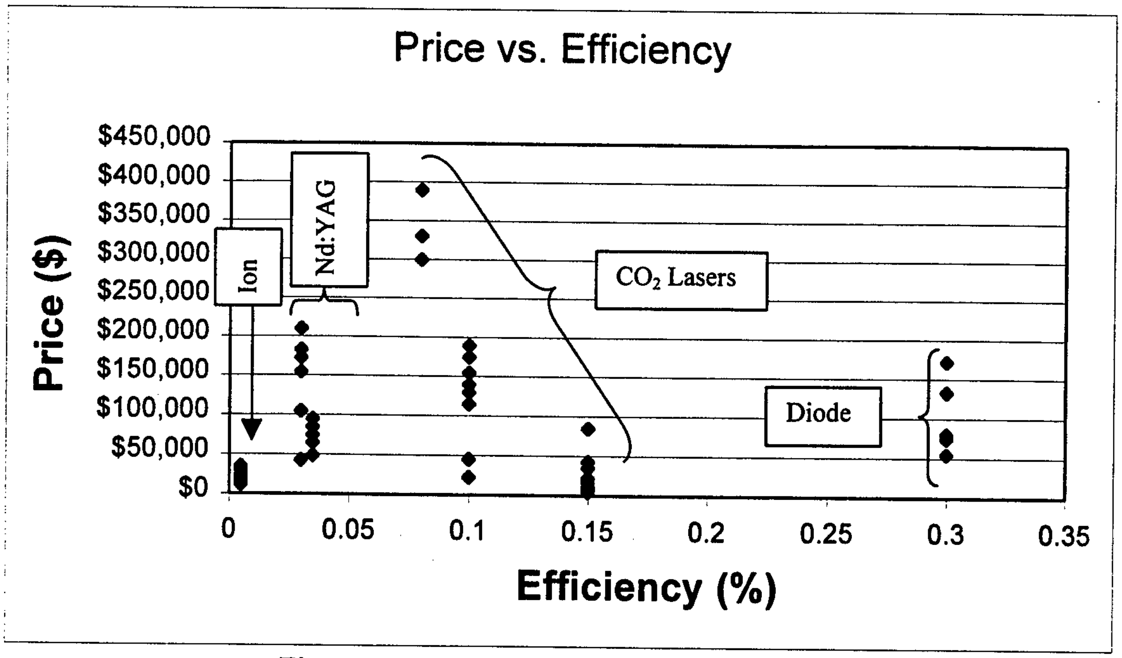


Figure 6. Scatter Plot of Price vs. Efficiency

We only had beam divergence data on CO_2 lasers and Ion lasers (See Figure 7). In both cases, there doesn't appear to be a strong relationship between beam divergence and cost. Logically, we would expect that price would tend to increase with a smaller beam divergence. The quicker a beam tends to spread, the less effective it becomes at further distances. This is another variable which, given our data, does not appear to be a major cost driver. There are CO_2 lasers with a beam divergence less than one (1) that range from $5,000 to $390,000. This leads us to believe that beam divergence is not a strong cost driver and; therefore, will not be considered further in our research.

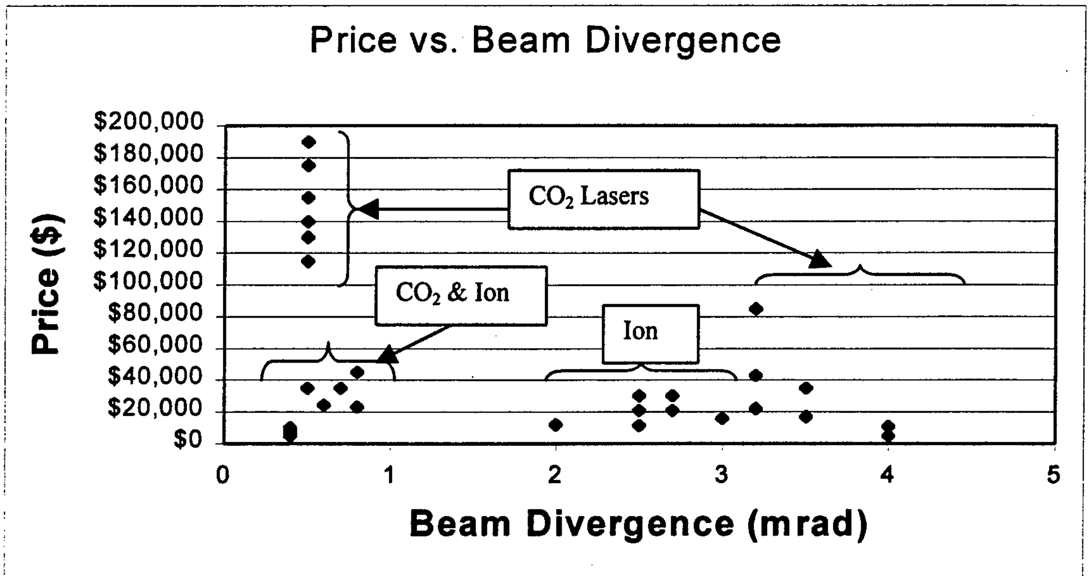


Figure 7. Scatter Plot of Price vs. Beam Divergence

Figure 8 shows a possible relationship between weight and cost. In general, as weight increases so does cost. It appears that the lighter a laser is, the less impact it has on cost. The jumble of lightweight lasers shown above in the bottom left-hand corner of the chart evidences this. Their cost tends to vary greatly. However, as lasers become much heavier, it seems to have an effect on cost. We will analyze this relationship in more depth. Weight may prove to be a valuable variable to include in our model.

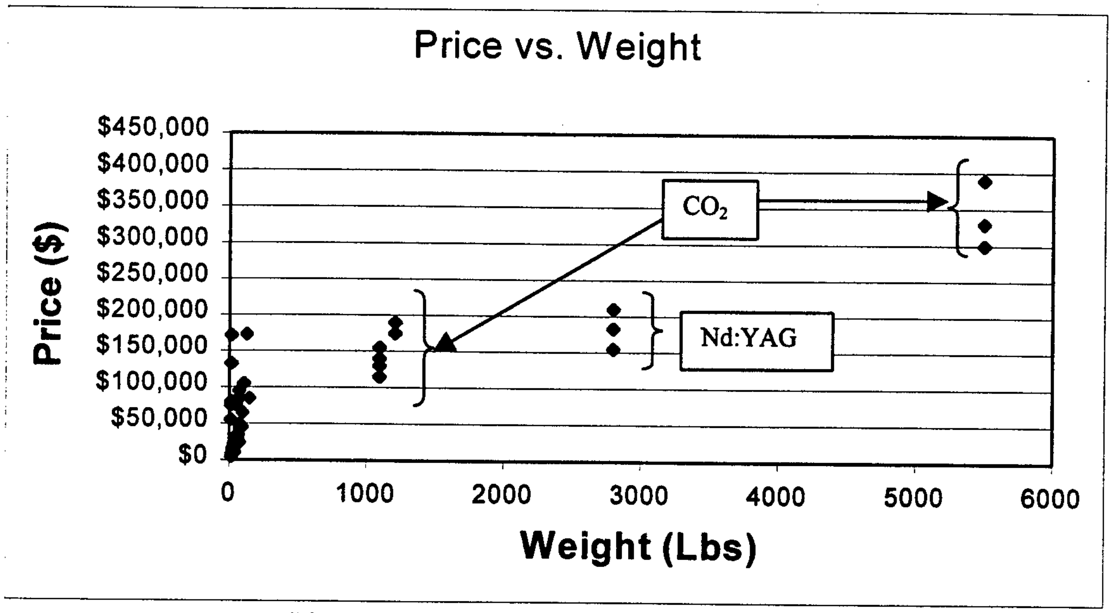


Figure 8. Scatter Plot of Price vs. Weight

Ordinary Least Squares Models

Using SAS™ statistical software, six models were developed to predict commercial laser prices. The logic of these models was based on the relationships identified in the scatter plots above.

Model 1 - PRICE v. POWER & LASER TYPE (CO_2, ION, YAG, & DIODE) (9)

A SAS™ LSBF program was used to determine the best fitting equation using the variables representing laser price, laser output power, and laser types. Table 4 below contains the equation along with key diagnostic measures. The excimer laser is represented by all zeros for this and all future models. Detailed statistical output and data plots are provided in Appendix B.

Model 1's SAS™ output indicates that the coefficient signs make logical sense for each variable. The coefficient for POWER is positive which means the PRICE of a laser device will increase as POWER increases. Meanwhile, depending on the laser type, there is an adjustment represented by the negative coefficients on the categorical variables.

With regard to the diagnostics, the equation *explains* 91.38% of the variation (R^2) with an adjusted R^2 only slightly lower at 90.45%. The large t-statistic scores and small p-values indicate that the variables are all significant above the 95% confidence level. The variance inflation factors (VIF) for each variable is lower than 10, indicating no problems with multicollinearity. A maximum VIF in excess of 10 is an indication of the presence of multicollinearity influencing the least squares estimates (Neter and others, 1996: 387). The equation's residual plots (Figure 9) indicate a potential problem with nonconstancy of error variance.

Table 4. Model 1 Statistical Summary

PRICE = 137,188 + 51.44 (POWER) – 112,312 (CO2) – 113,888 (ION) –59,336(YAG) – 78,840(DIODE)

Measures	Full Model	INTERCPT	POWER	CO2	ION	YAG	DIODE
Coefficient	--	137,188	51.44	-112,312	-113,888	-59,336	-78,840
R^2	.9138	--	--	--	--	--	--
R^2 Adj	.9045	--	--	--	--	--	--
F-Value	97.579	--	--	--	--	--	--
t-Stat	--	10.116	19.966	-7.340	-7.098	-3.772	-4.306
P-value	--	<.0001	<.0001	<.0001	<.0001	.0005	<.0001
VIF	--	--	1.19	3.99	2.83	3.11	2.06

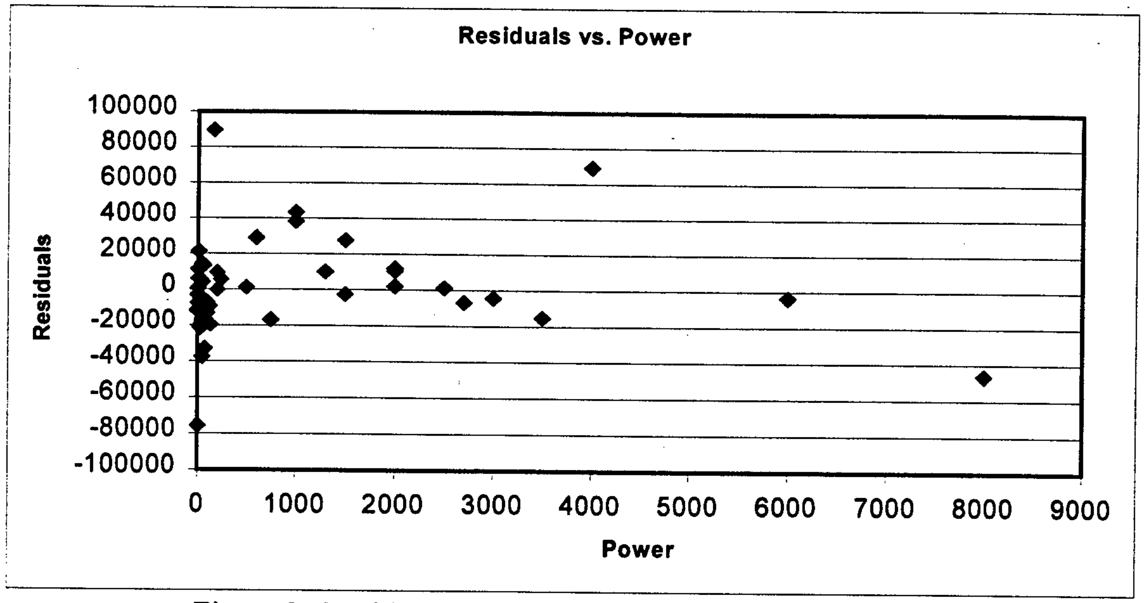


Figure 9. Residual Plot for Model 1 (Residuals vs. Power)

The overall statistics of this model are impressive; however, we still need to investigate our theory that price increases at a decreasing rate as power increases. Therefore, we performed a square root transformation of POWER for our next model.

Model 2 - PRICE v. SQRTPOW & LASER TYPE (CO_2, ION, YAG, & DIODE) (10)

Table 5 contains the equation along with key diagnostic measures. Detailed statistical output and data plots are provided in Appendix C.

This model had a slightly lower coefficient of multiple determination that the first model (.911 v .914) and the t-scores were still significant at the 95% confidence level. In addition, the signs of the coefficients still satisfy our logic. VIF scores are slightly higher than the previous model, but they are still significantly less than 10 so there's no concern with multicollinearity. In fact, just considering the statistical output, both models are

comparable. However, the residual plot for Model 2 doesn't appear to suffer from nonconstancy of error variance (See Figure 10).

Table 5. Model 2 Statistical Summary

PRICE = 110,815 + 3976.46 (SQRTPOW) – 125,566 (CO2) – 95,839 (ION) – 76,088 (YAG) - 106,480 (DIODE)

Measures	Full Model	Intercept	SQRTPOWER	CO2	ION	YAG	DIODE
Coefficient	--	110,815	3,976.46	-125,566	-95,839	-76,088	-106,480
R^2	.9110	--	--	--	--	--	--
R^2 Adj	.9014	--	--	--	--	--	--
F-Value	94.226	--	--	--	--	--	--
t-Stat	--	7.994	19.613	-7.980	-5.866	-4.729	-5.659
P-value	--	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
VIF	--	--	1.34	4.08	2.84	3.15	2.11

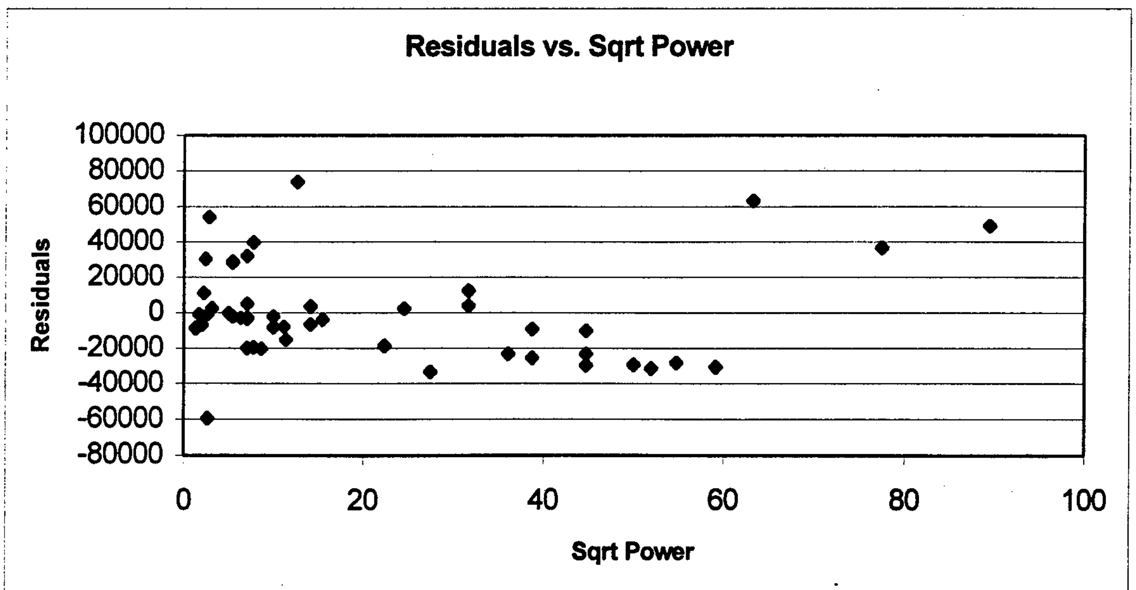


Figure 10. Residual Plot for Model 2 (Residuals vs. Sqrt Power)

Now that we've established that SQRTPOW is a beneficial variable, we will now try to improve on this model by adding WEIGHT to the equation. If you recall in our initial scatter plots, we discovered that POWER and WEIGHT were potential predictor variables.

Model 3 - PRICE v. SQRTPOW, WEIGHT, & LASER TYPE (CO_2, ION, YAG & DIODE) (11)

Table 6 contains the equation along with key diagnostic measures. Detailed statistical output and data plots are provided in Appendix D.

Table 6. Model 3 Statistical Summary

PRICE = 30,596 + 2,905.63(SQRTPOW) + 18.66(WEIGHT) – 34,047(CO2) – 14,285(ION) + 6,060.91(YAG)

Measures	Full Model	Intercept	SQRTPOW	WEIGHT	CO2	ION	YAG	DIODE
Coefficient	--	30,596	2,905.63	18.66	-34,047	-14,285	6,060.91	0
R^2	.9635	--	--	--	--	--	--	--
R^2 Adj	.9590	--	--	--	--	--	--	--
F-Value	216.293	--	--	--	--	--	--	--
t-Stat	--	3.018	11.715	5.219	-3.593	-1.248	0.579	--
P-value	--	.0044	<.0001	<.0001	.0009	.2191	.5657	--
VIF	--	--	4.43	3.83	3.18	3.15	2.82	--

With the addition of WEIGHT to the model, the ION, YAG and DIODE categorical variables lose their significance. Rather than eliminate all of these variables

in one step, we decided to eliminate DIODE because it was the least significant of the three. When we reran the model ION and YAG were still insignificant; therefore, we dropped YAG as well. We reran the model one more time without YAG and DIODE. The resulting equation is discussed below.

Model 4 - PRICE v. SQRTPOW, WEIGHT, & LASER TYPE (CO2 & ION) (12)

Table 7 contains the equation along with key diagnostic measures. Detailed statistical output and data plots are provided in Appendix E.

This model had roughly the same coefficient of multiple determination as the previous model; however, the t-scores for each variable proved to be significant at the 95% confidence level. In addition to significant t-scores, the sign of the coefficients satisfy our logic and the VIF scores were still below 10, indicating no problems with multicollinearity (See Figure 11).

Table 7. Model 4 Statistical Summary

PRICE = 35,377 + 2858.55 (SQRTPOW) + 19.39 (WEIGHT) – 38,239 (CO2) – 19,005(ION)

Measures	Full Model	Intercept	SQRTPOW	WEIGHT	CO2	ION
Coefficient	--	35,377	2,858.55	19.39	-38,239	-19,005
R^2	.9632	--	--	--	--	--
R^2 Adj	.9597	--	--	--	--	--
F-Value	274.63	--	--	--	--	--
t-Stat	--	6.061	12.297	5.855	-6.301	-2.382
P-value	--	<.0001	<.0001	<.0001	<.0001	.0218
VIF	--	--	3.96	3.34	1.32	1.55

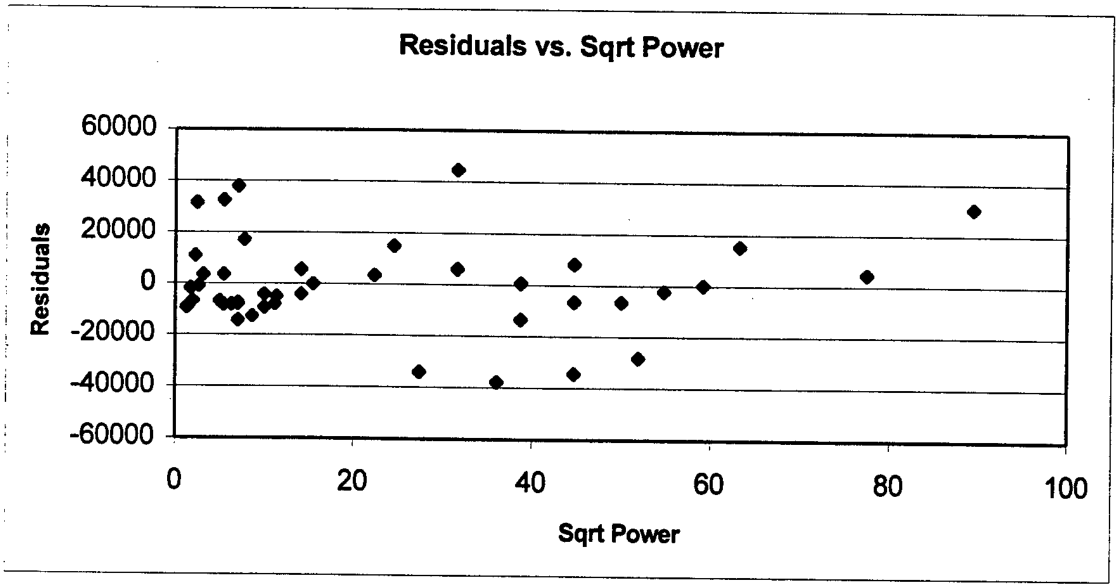


Figure 11. Residual Plot for Model 4 (Residuals vs. Sqrt Power)

After reviewing the plot of Residuals vs. WEIGHT, there appears to be an indication of interaction effects (See Figure 12). The data should appear random; however, there is a definite pattern. To investigate this phenomenon, we will run another model, which includes an interaction term between SQRTPOW and WEIGHT.

*Model 5 - PRICE v. SQRTPOW, WEIGHT, SQRTPOW*WEIGHT & LASER TYPE (CO2 & ION)* (13)

Table 8 contains the equation along with key diagnostic measures. Detailed statistical output and data plots are provided in Appendix F.

All the statistics in Table 8 look robust, with the exception of one. After adding the interaction term, WEIGHT becomes an insignificant variable with a p-value of 0.7571. There also seems to be a problem with multicollinearity, which is very likely when interaction terms are used. The VIF scores for WEIGHT and SQRTPOW are 18.20

and 16.70, respectively. We will now drop WEIGHT from the equation and run the model again.

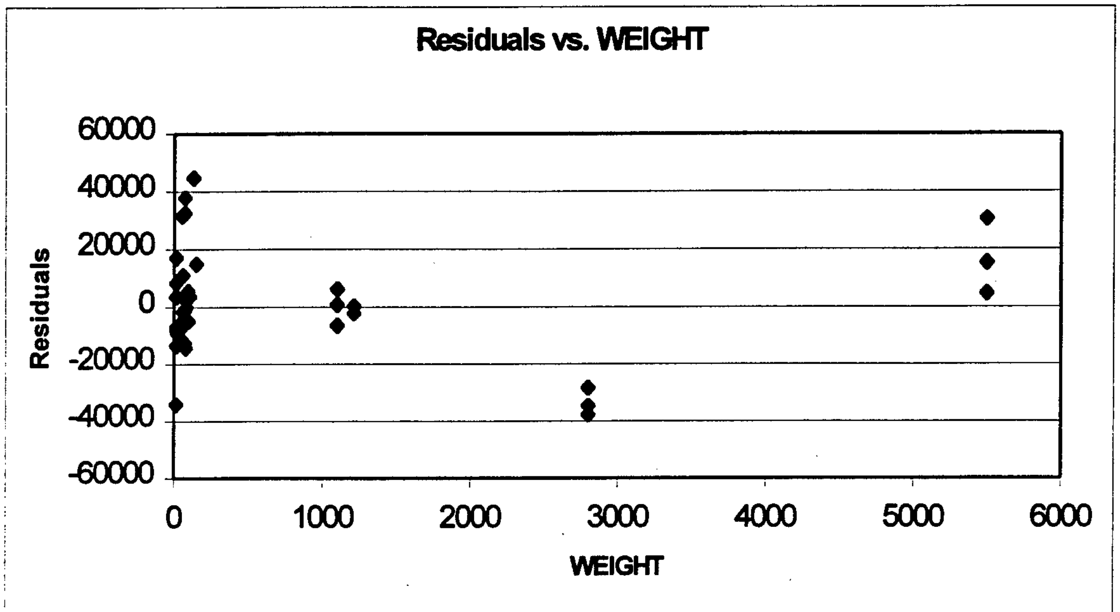


Figure 12. Residual Plot for Model 4 (Residuals vs. Weight)

Table 8. Model 5 Statistical Summary

PRICE = 41,218 + 2,815.23(SQRTPOW) – 2.144(WEIGHT) + 0.312(SQRTPOW*WEIGHT) – 41,833(CO2) - 23,660(ION)

Measures	Full Model	Intercept	SQRTPOW	WEIGHT	SQRTPOW*WT	CO2	ION
Coefficient	--	41,218	2,815.23	-2.144	0.312	41,833	-23,660
R^2	.9715	--	--	--	--	--	--
R^2 Adj	.9680	--	--	--	--	--	--
F-Value	279.543	--	--	--	--	--	--
t-Stat	--	7.544	13.577	-0.311	3.461	-7.603	-3.272
P-value	--	<.0001	<.0001	.7571	.0013	<.0001	.0022
VIF	--	--	3.97	18.20	16.70	1.38	1.61

*Model 6 - PRICE v. SQRTPOW, SQRTPOW*WEIGHT, & LASER TYPE (CO2 & ION):* (14)

Table 9 contains the equation along with key diagnostic measures. Detailed statistical output and data plots are provided in Appendix G.

Table 9. Model 6 Statistical Summary

PRICE = 41,017 + 2,796.13(SQRTPOW) + 0.286(SQRTPOW*WEIGHT) – 41,574(CO2) – 23,527(ION)

Measures	Full Model	Intercept	SQRTPOWER	SQRTPOW*WT	CO2	ION
Coefficient	--	41,017	2,796.13	0.286	-41,574	-23,527
R^2	.9714	--	--		--	--
R^2 Adj	.9687	--	--		--	--
F-Value	357.083	--	--		--	--
t-Stat	--	7.643	14.271	7.506	-7.727	-3.295
P-value	--	<.0001	<.0001	<.0001	<.0001	.0020
VIF	--	--	3.52	3.06	1.34	1.60

This model is the most robust and is the recommended model for predicting the price of lasers. However, depending on your view, there may be a problem with the model hierarchy. Model hierarchy implies that when there is an interaction term in an equation, the variables that make up that interaction term should also be in the model regardless of the statistical significance of the variables. If this is your view, refer back to equation 5. One thing that needs to be considered is that Model 5 (the hierarchical model) had problems with multicollinearity, which means the parameter estimates will be

more dependent on the data set you had and may not be stable. This reinforces our preference for Model 6.

The estimated regression function indicates that mean laser price is expected to increase by $2,796 when the square root of power increases by one watt, holding the other independent variables constant. The mean price is expected to increase by $0.29 when the interaction term of square root power and weight increase by one unit, holding the other variables constant. Finally, the mean price is expected to decrease by $41,574 when predicting the price of a CO_2 laser and decrease by $23,527 when predicting the price of an Ion laser, again holding all other variables constant. Below is a detailed analysis of the statistical output and plots.

Diagnostic Tests for Model #6

Coefficient of Multiple Determination (R^2). As mentioned earlier, the R^2 helps identify the amount of variation "explained" by the regression equation. We obtained an R^2 value of .9714. This score indicates that the regression equation in this model explains roughly 97% of the variation by using transformed (square root) power, square root of power times weight, and the qualitative variables for CO_2 and Ion lasers. Only 3% of the variation is due to random error or is explained by other omitted variables. SAS also provides an adjusted R^2 value. The adjusted R^2 of .9687 indicates that adjusting for the number of predictor variables in the model had only a small effect on R^2.

F-Test. The calculated F-value was 357.083. This is significantly greater than the table value of 2.63 [F(.95; 4, 42)], which leads us to believe, with 95% confidence, that

our model is appropriate as a predictor of laser price. We used Table B.4, Percentiles of the F Distribution, located in Appendix B of *Applied Linear Regression Models* (Neter and others, 1996:689-695).

t-Test. We conducted a *t*-test on each of the independent variables to determine the marginal effect each variable added to the model. The table value of t is 2.02 [$t(.975; 42)$]. We used Table B.2, Percentiles of the *t* Distribution, located in Appendix B of *Applied Linear Regression Models* (Neter and others, 1996:686–687). All the variables in our model exceeded 2.02, which confirms, with 95% confidence, that each variable adds statistical power to the model. The significance of the coefficients is further indicated by the p-values reported in Table 9 above.

The suitability of the regression equation was further scrutinized through graphical analysis and supporting statistical techniques. The following regression characteristics were examined: appropriateness of the regression model, constancy of error variance, normality of error terms, and presence of influential outlying data points.

Appropriateness of Regression Model. Our analysis of the appropriateness of the regression model included an examination of the residuals against the predicted values (See Figure 13 below). The plot does not suggest any systematic deviations from the response plane, or that the variance of the error terms varies with the level of the predicted values. To put another way, the data appears to be randomly plotted. This is indicative of an appropriate regression function.

Constancy of Error Variance. To test for constancy of error variance, the residuals were plotted against each of the independent variables (See Figures 14 & 15). Again, the randomness associated with these plots is consistent with the conclusion of

good fit by the response function and; therefore, we cannot reject that the error variance is constant.

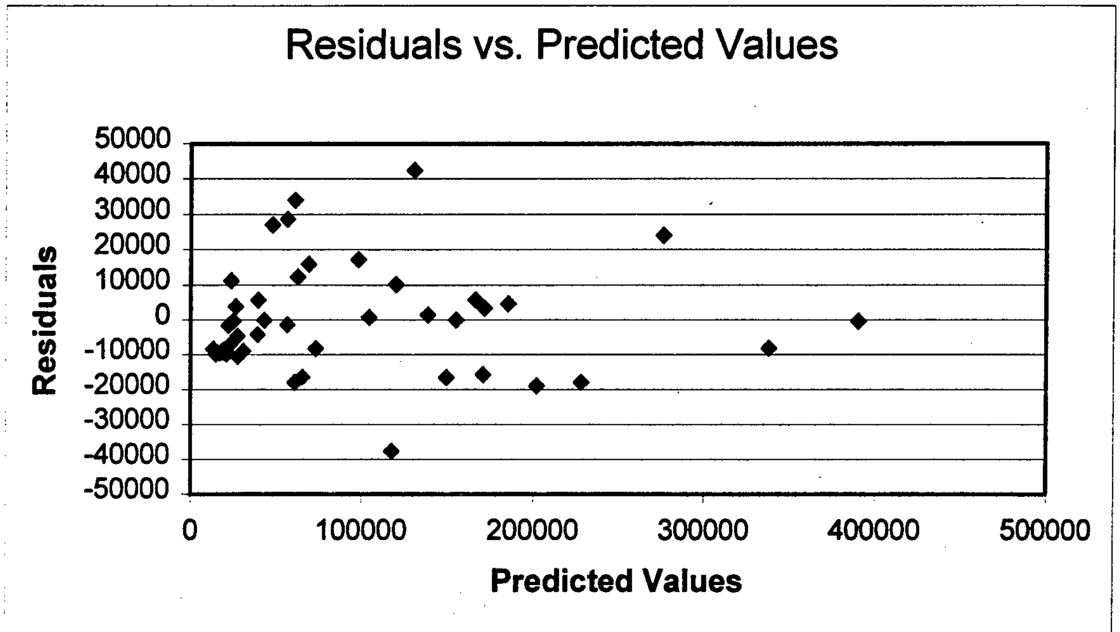


Figure 13. Plot of Residuals vs. Predicted Values

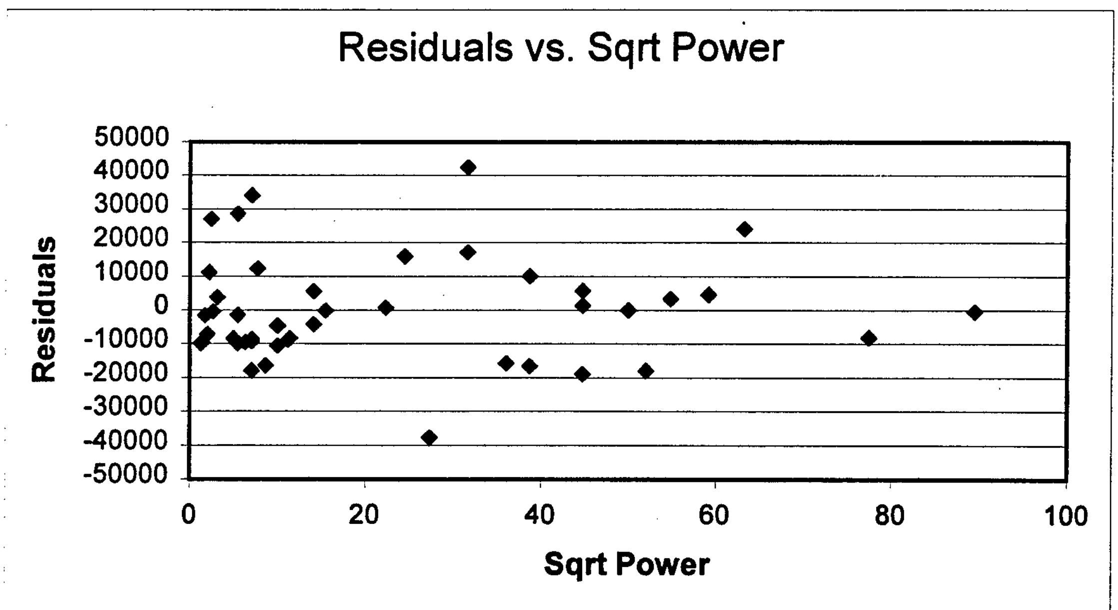


Figure 14. Residual Plot for Model 6 (Residuals vs. Sqrt Power)

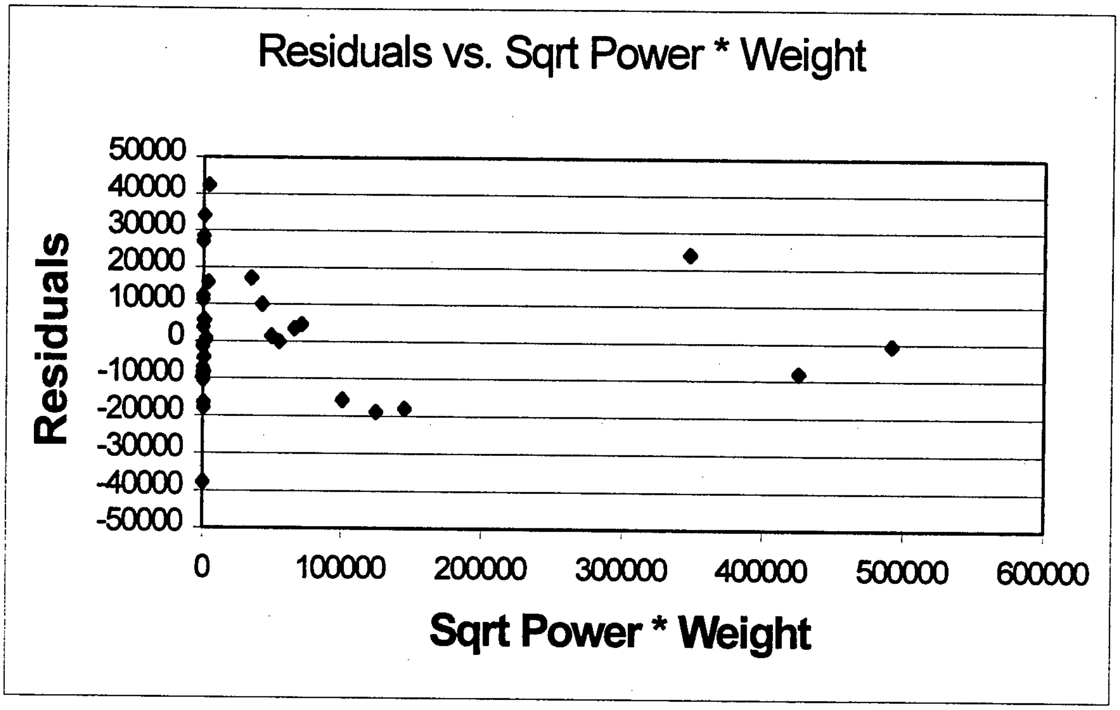


Figure 15. Residual Plot for Model 6 (Residuals vs. Sqrt Power*Weight)

Normality of Error Terms. A normal probability plot of the residuals was used to test for non-normality of error terms (Figure 16). While not perfectly straight, the residuals appeared to be nearly linear, suggesting agreement with normality. This plot also helps substantiate that there are no problems with nonconstancy of error variance, as the two tend to go hand-in-hand.

Influential Outlying Data Points. Finally, the data was examined for the presence of influential outlying data points by studying the studentized residuals, studentized deleted residuals, DFFITS, DFBETAS, and Cook's Distance. Each test is provided below:

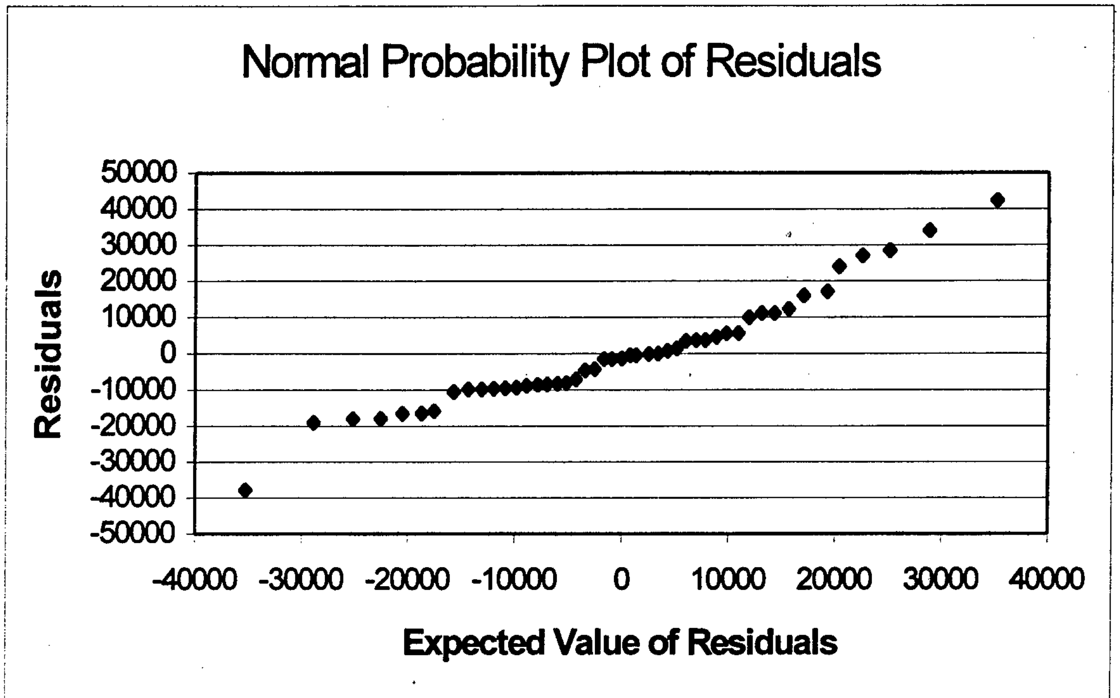


Figure 16. Normal Probability Plot of Residuals

Studentized Residuals. Observations are in terms of standard deviations and are considered outlying data points if the value is greater than the absolute value of 3.0. There were no data points that exceeded the critical value.

Studentized Deleted Residuals. Observations are in terms of standard deviations and are considered outlying data points if the value is greater than the absolute value of 3.0. Observation 44 had a studentized deleted residual value of 3.0765, which slightly exceeds the critical value. This observation was analyzed using the techniques described below to determine if it is influential. It was determined not to be influential; therefore, it will be left in the data set.

DFFITS. For the size of our data set, the critical DFFITS value is 1.0. This means that any value greater than 1.0 may be an influential outlier and needs to be

investigated further. There were no data points with DFFITS values that exceeded the critical value.

DFBETAS. The critical DFBETA value is also 1.0 and any data point with a DFBETA that exceeds this value needs to be investigated further as it is considered influential. Again, there were no data points that exceeded the critical value.

Cook's Distance. This is an aggregate measure of influence, which shows the effect of a particular data point on all fitted values. To determine the critical value for Cook's Distance, we used the 50^{th} percentile of the F-distribution [$F(4, 43)$]. This value was determined to be 0.854. Any observations with Cook's D values that exceed the critical value are considered influential and should be investigated further. No data points exceeded the critical value.

Using DFFITS, DFBETAS, and Cook's Distance, it was determined that there were no influential outlying data points in our database.

V. Findings and Recommendations

The increase in the use of laser technology in military systems in the DoD has created the need to develop a methodology to estimate the costs of these new proposals. While there are some existing laser systems, cost data on these programs is scarce. As the DoD moves toward more commercial acquisition practices and initiatives, the collection of future detailed laser cost data will be less likely.

This study attempts to estimate the price of a laser device using the technique of ordinary least squares regression on a database populated by commercial laser parameters to include price. Using this technique, six models were developed with various statistical results. The final model is the most statistically robust and would be useful to help estimate the production price of a proposed laser device. A Work Breakdown Structure (WBS) is also provided as a framework for use in determining the other cost categories needed to estimate the full military laser system.

Findings

Using multivariate ordinary least squares regression, six models were developed using a commercial laser database. The equations were subjected to a variety of diagnostic tests and all demonstrated robust results.

Model #6 had the most robust and significant results. This equation is as follows:

$$\text{PRICE} = 41{,}017 + 2{,}796.13(\text{SQRTPOW}) + 0.286(\text{SQRTPOW}*\text{WEIGHT}) - 41{,}574(\text{CO2}) - 23{,}527(\text{ION})$$

As a reminder, the unit of measure for the SQRTPOW variable is watts and the SQRTPOW*WEIGHT variable is watt-pounds.

The statistical data clearly suggest that there is a strong relationship between our model and the price of lasers. The above equation had a coefficient of multiple determination (R^2) of .97 and an adjusted coefficient of multiple determination (Adj. R^2) of .96. The F score on this model was very significant at 357. All of the variable coefficients, except ION, were significant to less than .0001. The ION coefficient was significant to .0020. In addition, the model showed a low degree of multicollinearity as measured by the coefficient variance inflation factors (VIF) being 3.52 or below.

All of the models could be useful to an estimator who is trying to predict the price of a laser device. However, the model chosen for use is dependent on the information in hand. The estimator's knowledge of the project's output power, laser type, and laser device weight will determine which equation is most suited for use. Another factor in choosing between Model #5 and Model #6 is the reader's belief in model hierarchy. If the reader feels that a model with an interaction term should also include the variables that make up that interaction term, regardless of the fact that one of those variables is not statistically significant, then Model #5 is the best choice. As mentioned earlier, Model #5 had problems with multicollinearity, meaning that the parameter estimates will be more dependent on the data set you had and may not be stable. We recommend the use of Model #6.

It's important to realize that the CER estimates the average price of a laser with certain parameters. It's still possible to see variances between vendors. For example, the root MSE of Model #6 is almost $16K, so a reasonable confidence interval around the

mean could be wider than $64K, with a prediction interval on any single system being even wider. The width of these intervals depends on the distance of the estimated device's performance parameters from the *center* of the data [see Neter and others, Section 6.7 for guidance on how to estimate these interval widths].

Assumptions and Limitations

Several assumptions and limitations were recognized during our analysis. First, all data points were assumed accurate. In other words, no errors in recording or miscalculations were made in gathering the data. We also assumed that the accounting systems for collected price data were similar for all laser types. In addition, we assumed that the causation system remained constant for all data points. Finally, a 95% confidence level or a 5% significance level was chosen as the standard for all data analysis.

The models are limited by the historical data being used to develop the model. Logically, there is a point in performance characteristic that is limited by technology. This research uses lasers that were developed using established technology. Based on this, our model estimates prices that are increasing at a decreasing rate as the independent variables increase. As requirements push the envelope of technology, we expect the price relationship to change to one that increases at an increasing rate. Therefore, the models are most accurate within the relevant range of variables in the database. Outside the database relevant range, the equations become less reliable because relationship of the variables can not be guaranteed outside this range.

Another limiting factor in our research is that the majority of points in our database were of low to medium power. We only had three data points greater than or equal to 4,000 watts. This limits our ability to make inferences regarding the behavior of higher-powered lasers.

Recommendations for Further Study

The DoD is heading toward the use of more commercial based acquisition practices and has initiated the use of price-based acquisition. Therefore, there seems to be significant value in further research in several areas.

One area of further study is to improve the commercial laser database in Appendix A. The laser database could be expanded to include more diverse laser types, power ranges, and weights. The larger depth and breadth of the database would increase the relevant range in which any laser parametric equations could be used with confidence.

A second area of further study would be an investigation into the use of other cost estimating techniques, such as analogous or expert judgement, to improve upon the predictive ability of the parametric method used in this effort.

A final area of study is the development of a factor set to estimate the cost of a total military laser system. The equations developed in this effort are useful in the development of a single production laser device. This effort includes a laser Work Breakdown Structure (WBS) as a framework to categorize and collect the other costs that could make up a total military laser system. With the WBS as a foundation, a further effort would seek to develop a set of cost factors. These cost factors could then be used to build upon the laser device estimate calculated by this effort's set of equations.

Summary

This thesis built a commercial laser database containing commercial laser parameters and prices gathered from commercial manufacturer catalogs. Laser cost drivers were analyzed and evaluated to determine potential candidates for model inclusion. An ordinary least squares (multivariate least squares) regression method was used and several diagnostic tests were performed to develop suitable explanatory models based on the commercial laser database. Six models were developed based on the regression with one model being particularly robust. Our hope is that this research will be beneficial for future cost estimates—at the very least, a foundation for future research.

Appendix A. Data Set

Laser Type	Price	Output Power (W)	CO2	ION	YAG	DIODE	Wavelength (nm)	Efficiency	Weight (Lbs)	Beam Divergence (mrad)
CO2	$5,000	30	1	0	0	0	10600	0.15	18	0.4
CO2	$7,600	40	1	0	0	0	10600	0.15	22	0.4
CO2	$10,000	50	1	0	0	0	10600	0.15	26	0.4
Argon Ion	$11,300	1.6	0	1	0	0	488	0.005	26.4	2.5
Argon Ion	$12,000	2	0	1	0	0	488	0.005	26.4	2
Argon Ion	$16,000	4	0	1	0	0	488	0.005	26.4	3
CO2	$5,000	25	1	0	0	0	10,600	0.15	18	4
CO2	$10,780	50	1	0	0	0	10,600	0.15	44	4
CO2	$17,000	100	1	0	0	0	10,590	0.15	30	3.5
CO2	$22,000	125	1	0	0	0	10,590	0.15	36	3.2
CO2	$35,000	200	1	0	0	0	10,590	0.15	70	3.5
CO2	$43,000	240	1	0	0	0	10,590	0.15	84	3.2
CO2	$85,000	600	1	0	0	0	10,590	0.15	150	3.2
Argon Ion	$24,500	7	0	1	0	0	488	0.005	77	0.6
Argon Ion	$30,100	10	0	1	0	0	488	0.005	61	2.7
Krypton Ion	$30,100	10	0	1	0	0	647.1	0.005	61	2.5
Argon Ion	$20,850	3	0	1	0	0	488	0.005	61	2.7
Krypton Ion	$20,850	3	0	1	0	0	647.1	0.005	61	2.5
Argon Ion	$34,950	5	0	1	0	0	488	0.005	62	0.5
Krypton Ion	$34,950	5	0	1	0	0	514.5	0.005	62	0.7
CO2	$23,000	100	1	0	0	0	10,640	0.10	70.4	0.8
CO2	$45,000	200	1	0	0	0	10,640	0.10	95.7	0.8
CO2	$115,000	1000	1	0	0	0	10,640	0.10	1100	0.5
CO2	$130,000	1500	1	0	0	0	10,640	0.10	1100	0.5
CO2	$140,000	2000	1	0	0	0	10,640	0.10	1100	0.5
CO2	$155,000	2500	1	0	0	0	10,640	0.10	1100	0.5
CO2	$175,000	3000	1	0	0	0	10,640	0.10	1210	0.5
CO2	$190,000	3500	1	0	0	0	10,640	0.10	1210	0.5
Nd:YAG	$75,000	6	0	0	1	0	1064	0.035	55	
Nd:YAG	$85,000	30	0	0	1	0	1064	0.035	75	
Nd:YAG	$95,000	50	0	0	1	0	1064	0.035	75	
Nd:YAG	$49,000	75	0	0	1	0	1064	0.035	75	
Nd:YAG	$65,000	130	0	0	1	0	1064	0.035	100	
Diode	$55,000	30	0	0	0	1	940	0.3	15	
Diode	$75,000	60	0	0	0	1	940	0.3	15	
Diode	$80,000	750	0	0	0	1	940	0.3	17.6	
Diode	$133,000	1500	0	0	0	1	940	0.3	17.6	
Diode	$172,000	2000	0	0	0	1	940	0.3	17.6	
Nd:YAG	$155,000	1300	0	0	1	0	1064	0.03	2800	
Nd:YAG	$183,000	2000	0	0	1	0	1064	0.03	2800	
Nd:YAG	$210,000	2700	0	0	1	0	1064	0.03	2800	
Nd:YAG	$43,000	50	0	0	1	0	1064	0.03	80	
Nd:YAG	$105,000	500	0	0	1	0	1064	0.03	110	
Nd:YAG	$173,000	1000	0	0	1	0	1064	0.03	130	
CO2	$300,000	4000	1	0	0	0	10,640	0.08	5500	
CO2	$330,000	6000	1	0	0	0	10,640	0.08	5500	
CO2	$390,000	8000	1	0	0	0	10,640	0.08	5500	
Excimer	$62,000	7	0	0	0	0	248			
Excimer	$122,000	60	0	0	0	0	248			
Excimer	$144,000	50	0	0	0	0	248			
Excimer	$235,000	160	0	0	0	0	248			
Nd:YAG	$100,000	8	0	0	1	0	1064			
Excimer	$94,000		0	0	0	0	248			

Appendix B. SAS Output for Model 1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	358867933902	71773586780	97.579	0.0001
Error	46	33835029106	735544110.99		
C Total	51	392702963008			

Root MSE	27120.91649	R-square	0.9138
Dep Mean	93941.92308	Adj R-sq	0.9045
C.V.	28.86988		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	137188	13561.632008	10.116	0.0001
POWER	1	51.443211	2.57651408	19.966	0.0001
CO2	1	-112312	15301.625984	-7.340	0.0001
ION	1	-113888	16045.802925	-7.098	0.0001
YAG	1	-59336	15730.604278	-3.772	0.0005
DIODE	1	-78840	18309.292049	-4.306	0.0001

Variable	DF	Standardized Estimate	Tolerance
INTERCEP	1	0.00000000	.
POWER	1	0.94351528	0.83876011
CO2	1	-0.63413884	0.25093192
ION	1	-0.51649761	0.35370392
YAG	1	-0.28767556	0.32201790
DIODE	1	-0.26745348	0.48551326

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	26418.5	7145.154	-21418.5	26162.78	-0.819
2	7600.0	26932.9	7130.749	-19332.9	26166.71	-0.739
3	10000.0	27447.3	7116.409	-17447.3	26170.61	-0.667
4	11300.0	23382.0	8576.391	-12082.0	25729.16	-0.470
5	12000.0	23402.6	8576.390	-11402.6	25729.16	-0.443
6	16000.0	23505.5	8576.387	-7505.5	25729.16	-0.292
7	5000.0	26161.3	7152.380	-21161.3	26160.80	-0.809
8	10780.0	27447.3	7116.409	-16667.3	26170.61	-0.637
9	17000.0	30019.5	7045.684	-13019.5	26189.74	-0.497
10	22000.0	31305.6	7010.942	-9305.6	26199.06	-0.355
11	35000.0	35163.8	6909.271	-163.8	26226.06	-0.006
12	43000.0	37221.6	6856.656	5778.4	26239.86	0.220
13	85000.0	55741.1	6438.254	29258.9	26345.64	1.111
14	24500.0	23659.8	8576.388	840.2	25729.16	0.033
15	30100.0	23814.1	8576.396	6285.9	25729.16	0.244
16	30100.0	23814.1	8576.396	6285.9	25729.16	0.244
17	20850.0	23454.0	8576.388	-2604.0	25729.16	-0.101
18	20850.0	23454.0	8576.388	-2604.0	25729.16	-0.101
19	34950.0	23556.9	8576.387	11393.1	25729.16	0.443
20	34950.0	23556.9	8576.387	11393.1	25729.16	0.443
21	23000.0	30019.5	7045.684	-7019.5	26189.74	-0.268
22	45000.0	35163.8	6909.271	9836.2	26226.06	0.375
23	115000	76318.4	6106.432	38681.6	26424.53	1.464
24	130000	102040	5922.207	27960.0	26466.42	1.056
25	140000	127762	6014.631	12238.4	26445.57	0.463
26	155000	153483	6371.675	1516.8	26361.83	0.058
27	175000	179205	6952.690	-4204.8	26214.58	-0.160
28	190000	204926	7707.190	-14926.4	26002.76	-0.574
29	75000.0	78160.5	8005.221	-3160.5	25912.56	-0.122
30	85000.0	79395.1	7992.551	5604.9	25916.47	0.216
31	95000.0	80424.0	7982.344	14576.0	25919.61	0.562
32	49000.0	81710.1	7970.035	-32710.1	25923.40	-1.262
33	65000.0	84539.5	7944.726	-19539.5	25931.17	-0.754
34	55000.0	59890.6	12319.52	-4890.6	24161.41	-0.202
35	75000.0	61433.9	12306.21	13566.1	24168.19	0.561
36	80000.0	96929.7	12132.65	-16929.7	24255.78	-0.698
37	133000	135512	12237.66	-2512.1	24202.97	-0.104
38	172000	161234	12474.59	10766.3	24081.71	0.447
39	155000	144728	8004.058	10272.0	25912.91	0.396
40	183000	180738	8562.756	2261.7	25733.70	0.088

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	216749	9438.342	-6748.5	25425.61	-0.265
42	43000.0	80424.0	7982.344	-37424.0	25919.61	-1.444
43	105000	103573	7839.193	1426.5	25963.27	0.055
44	173000	129295	7879.701	43704.9	25951.00	1.684
45	300000	230648	8589.580	69352.0	25724.76	2.696
46	330000	333534	12825.50	-3534.4	23896.67	-0.148
47	390000	436421	17558.85	-46420.9	20669.56	-2.246
48	62000.0	137548	13561.41	-75547.7	23486.86	-3.217
49	122000	140274	13560.48	-18274.2	23487.39	-0.778
50	144000	139760	13560.55	4240.3	23487.35	0.181
51	235000	145418	13562.47	89581.5	23486.24	3.814
52	100000	78263.4	8004.148	21736.6	25912.89	0.839

Obs	-2-1 -0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
1	| *| |	0.008	-0.8157	0.0694	1.1227	-0.2228
2	| *| |	0.007	-0.7351	0.0691	1.1410	-0.2003
3	| *| |	0.005	-0.6626	0.0689	1.1560	-0.1802
4	| | |	0.004	-0.4656	0.1000	1.2317	-0.1552
5	| | |	0.004	-0.4393	0.1000	1.2356	-0.1464
6	| | |	0.002	-0.2888	0.1000	1.2537	-0.0963
7	| *| |	0.008	-0.8058	0.0695	1.1252	-0.2203
8	| *| |	0.005	-0.6327	0.0689	1.1619	-0.1720
9	| | |	0.003	-0.4930	0.0675	1.1846	-0.1326
10	| | |	0.002	-0.3518	0.0668	1.2027	-0.0941
11	| | |	0.000	-0.0062	0.0649	1.2201	-0.0016
12	| | |	0.001	0.2179	0.0639	1.2112	0.0569
13	| |** |	0.012	1.1135	0.0564	1.0272	0.2721
14	| | |	0.000	0.0323	0.1000	1.2676	0.0108
15	| | |	0.001	0.2418	0.1000	1.2579	0.0806
16	| | |	0.001	0.2418	0.1000	1.2579	0.0806
17	| | |	0.000	-0.1001	0.1000	1.2660	-0.0334
18	| | |	0.000	-0.1001	0.1000	1.2660	-0.0334
19	| | |	0.004	0.4389	0.1000	1.2357	0.1463
20	| | |	0.004	0.4389	0.1000	1.2357	0.1463
21	| | |	0.001	-0.2653	0.0675	1.2121	-0.0714
22	| | |	0.002	0.3715	0.0649	1.1979	0.0979
23	| |** |	0.019	1.4828	0.0507	0.9027	0.3427
24	| |** |	0.009	1.0578	0.0477	1.0339	0.2367
25	| | |	0.002	0.4588	0.0492	1.1668	0.1043

<table>
<tr><th>Obs</th><th>-2 -1 -0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>26</td><td>| | |</td><td>0.000</td><td>0.0569</td><td>0.0552</td><td>1.2071</td><td>0.0138</td></tr>
<tr><td>27</td><td>| | |</td><td>0.000</td><td>-0.1587</td><td>0.0657</td><td>1.2171</td><td>-0.0421</td></tr>
<tr><td>28</td><td>| *| |</td><td>0.005</td><td>-0.5698</td><td>0.0808</td><td>1.1888</td><td>-0.1689</td></tr>
<tr><td>29</td><td>| | |</td><td>0.000</td><td>-0.1207</td><td>0.0871</td><td>1.2474</td><td>-0.0373</td></tr>
<tr><td>30</td><td>| | |</td><td>0.001</td><td>0.2140</td><td>0.0868</td><td>1.2419</td><td>0.0660</td></tr>
<tr><td>31</td><td>| |* |</td><td>0.005</td><td>0.5581</td><td>0.0866</td><td>1.1985</td><td>0.1719</td></tr>
<tr><td>32</td><td>| **| |</td><td>0.025</td><td>-1.2702</td><td>0.0864</td><td>1.0109</td><td>-0.3905</td></tr>
<tr><td>33</td><td>| *| |</td><td>0.009</td><td>-0.7499</td><td>0.0858</td><td>1.1584</td><td>-0.2298</td></tr>
<tr><td>34</td><td>| | |</td><td>0.002</td><td>-0.2003</td><td>0.2063</td><td>1.4299</td><td>-0.1021</td></tr>
<tr><td>35</td><td>| |* |</td><td>0.014</td><td>0.5571</td><td>0.2059</td><td>1.3787</td><td>0.2837</td></tr>
<tr><td>36</td><td>| *| |</td><td>0.020</td><td>-0.6940</td><td>0.2001</td><td>1.3382</td><td>-0.3471</td></tr>
<tr><td>37</td><td>| | |</td><td>0.000</td><td>-0.1027</td><td>0.2036</td><td>1.4306</td><td>-0.0519</td></tr>
<tr><td>38</td><td>| | |</td><td>0.009</td><td>0.4432</td><td>0.2116</td><td>1.4098</td><td>0.2296</td></tr>
<tr><td>39</td><td>| | |</td><td>0.002</td><td>0.3927</td><td>0.0871</td><td>1.2244</td><td>0.1213</td></tr>
<tr><td>40</td><td>| | |</td><td>0.000</td><td>0.0869</td><td>0.0997</td><td>1.2660</td><td>0.0289</td></tr>
<tr><td>41</td><td>| | |</td><td>0.002</td><td>-0.2627</td><td>0.1211</td><td>1.2863</td><td>-0.0975</td></tr>
<tr><td>42</td><td>| **| |</td><td>0.033</td><td>-1.4616</td><td>0.0866</td><td>0.9457</td><td>-0.4501</td></tr>
<tr><td>43</td><td>| | |</td><td>0.000</td><td>0.0543</td><td>0.0835</td><td>1.2445</td><td>0.0164</td></tr>
<tr><td>44</td><td>| |*** |</td><td>0.044</td><td>1.7196</td><td>0.0844</td><td>0.8506</td><td>0.5221</td></tr>
<tr><td>45</td><td>| |**** |</td><td>0.135</td><td>2.9059</td><td>0.1003</td><td>0.4519</td><td>0.9703</td></tr>
<tr><td>46</td><td>| | |</td><td>0.001</td><td>-0.1463</td><td>0.2236</td><td>1.4654</td><td>-0.0785</td></tr>
<tr><td>47</td><td>| ****| |</td><td>0.607</td><td>-2.3541</td><td>0.4192</td><td>0.9786</td><td>-1.9998</td></tr>
<tr><td>48</td><td>|*****| |</td><td>0.575</td><td>-3.6137</td><td>0.2500</td><td>0.3298</td><td>-2.0866</td></tr>
<tr><td>49</td><td>| *| |</td><td>0.034</td><td>-0.7747</td><td>0.2500</td><td>1.4051</td><td>-0.4472</td></tr>
<tr><td>50</td><td>| | |</td><td>0.002</td><td>0.1786</td><td>0.2500</td><td>1.5148</td><td>0.1031</td></tr>
<tr><td>51</td><td>| |*****|</td><td>0.809</td><td>4.5624</td><td>0.2501</td><td>0.1554</td><td>2.6346</td></tr>
<tr><td>52</td><td>| |* |</td><td>0.011</td><td>0.8361</td><td>0.0871</td><td>1.1394</td><td>0.2583</td></tr>
</table>

Obs	INTERCEP Dfbetas	POWER Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas	DIODE Dfbetas
1	-0.0016	0.1248	-0.1032	0.0013	-0.0120	-0.0140
2	-0.0015	0.1117	-0.0928	0.0012	-0.0107	-0.0126
3	-0.0013	0.1001	-0.0835	0.0010	-0.0096	-0.0112
4	-0.0000	0.0002	-0.0000	-0.0829	-0.0000	-0.0000
5	-0.0000	0.0001	-0.0000	-0.0783	-0.0000	-0.0000
6	-0.0000	0.0000	-0.0000	-0.0515	-0.0000	-0.0000
7	-0.0016	0.1237	-0.1021	0.0013	-0.0119	-0.0139
8	-0.0013	0.0955	-0.0797	0.0010	-0.0092	-0.0107
9	-0.0009	0.0720	-0.0614	0.0007	-0.0069	-0.0081
10	-0.0007	0.0505	-0.0436	0.0005	-0.0048	-0.0057
11	-0.0000	0.0008	-0.0008	0.0000	-0.0001	-0.0001
12	0.0004	-0.0288	0.0263	-0.0003	0.0028	0.0032
13	0.0014	-0.1071	0.1241	-0.0011	0.0103	0.0120
14	-0.0000	0.0000	-0.0000	0.0058	-0.0000	-0.0000
15	-0.0000	0.0001	-0.0000	0.0431	-0.0000	-0.0000
16	-0.0000	0.0001	-0.0000	0.0431	-0.0000	-0.0000
17	-0.0000	0.0000	-0.0000	-0.0178	-0.0000	-0.0000
18	-0.0000	0.0000	-0.0000	-0.0178	-0.0000	-0.0000
19	0.0000	-0.0000	0.0000	0.0782	0.0000	0.0000
20	0.0000	-0.0000	0.0000	0.0782	0.0000	0.0000
21	-0.0005	0.0387	-0.0331	0.0004	-0.0037	-0.0044
22	0.0007	-0.0505	0.0453	-0.0005	0.0048	0.0057
23	0.0011	-0.0844	0.1500	-0.0009	0.0081	0.0095
24	0.0001	-0.0086	0.0937	-0.0001	0.0008	0.0010
25	-0.0002	0.0186	0.0350	0.0002	-0.0018	-0.0021
26	-0.0001	0.0051	0.0036	0.0001	-0.0005	-0.0006
27	0.0003	-0.0221	-0.0082	-0.0002	0.0021	0.0025
28	0.0014	-0.1082	-0.0226	-0.0011	0.0104	0.0122
29	-0.0001	0.0078	-0.0020	0.0001	-0.0189	-0.0009
30	0.0002	-0.0133	0.0034	-0.0001	0.0334	0.0015
31	0.0004	-0.0335	0.0085	-0.0003	0.0871	0.0038
32	-0.0010	0.0731	-0.0186	0.0008	-0.1979	-0.0082
33	-0.0005	0.0391	-0.0100	0.0004	-0.1164	-0.0044
34	-0.0002	0.0179	-0.0046	0.0002	-0.0017	-0.0686
35	0.0006	-0.0480	0.0122	-0.0005	0.0046	0.1906
36	-0.0001	0.0087	-0.0022	0.0001	-0.0008	-0.2309
37	0.0001	-0.0069	0.0018	-0.0001	0.0007	-0.0333
38	-0.0007	0.0537	-0.0137	0.0006	-0.0051	0.1418
39	-0.0003	0.0252	-0.0064	0.0003	0.0566	-0.0028
40	-0.0002	0.0117	-0.0030	0.0001	0.0120	-0.0013

Obs	INTERCEP Dfbetas	POWER Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas	DIODE Dfbetas
41	0.0007	-0.0545	0.0139	-0.0006	-0.0350	0.0061
42	-0.0012	0.0878	-0.0224	0.0009	-0.2281	-0.0099
43	0.0000	-0.0008	0.0002	-0.0000	0.0082	0.0001
44	-0.0008	0.0591	-0.0151	0.0006	0.2525	-0.0066
45	-0.0093	0.7032	0.0792	0.0072	-0.0674	-0.0790
46	0.0009	-0.0697	0.0038	-0.0007	0.0067	0.0078
47	0.0248	-1.8828	0.2195	-0.0194	0.1804	0.2116
48	-2.0866	0.0247	1.8427	1.7635	1.7962	1.5425
49	-0.4472	0.0008	0.3962	0.3780	0.3855	0.3312
50	0.1031	-0.0004	-0.0913	-0.0872	-0.0889	-0.0763
51	2.6334	0.0454	-2.3460	-2.2257	-2.2751	-1.9561
52	0.0007	-0.0537	0.0137	-0.0006	0.1309	0.0060

```
                          Partial Regression Residual Plot
        ---+----+----+----+----+----+----+----+----+----+----+----+-----
 PRICE |                                                              |
 400000+                                                              +
       |                                                              |
       |                                                              |
 200000+                                                            1 +
       |                                                            2 |
       |        3                                                   1 |
      0+       1*                                                     +
       |       11                                                     |
       |                                                              |
-200000+                                                              +
       |                                                              |
        ---+----+----+----+----+----+----+----+----+----+----+----+-----
         -0.1  0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0

                                   INTERCEP
```

```
                         Partial Regression Residual Plot
       +------+------+------+------+------+------+------+------+------+----+--
 PRICE |                                                                    |
400000+                                                                     +
       |                                                                    |
       |                                                                   1|
200000+                                  1             1                    +
       |                                                                    |
       |               111 11 111   11                                      |
     0+         1113  3*   1                                                +
       |     351  113   1                                                   |
       |     2                                                              |
-200000+                                                                    +
       |                                                                    |
       +------+------+------+------+------+------+------+------+------+----+--
     -2000  -1000    0    1000   2000   3000   4000   5000   6000  7000

                                    POWER
```

```
                         Partial Regression Residual Plot
        ---+----+----+----+----+----+----+----+----+----+----+----+-----
  PRICE |                                                              |
 200000+                                                               +
        |       1                                                      |
        |                                                              |
 100000+      1                                                        +
        |       1                                     1                |
        |                                         1 1       1          |
      0+      1                               1122*32 1 111 2          +
        |                                    1     13 1       28       |
        |                                                              |
-100000+                                                               +
        |                                                              |
        ---+----+----+----+----+----+----+----+----+----+----+----+-----
         -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3

                                     CO2
```

```
                        Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |         1                                                    |
       |                                                              |
 100000+         1                                                    +
       |         1                                   1                |
       |                                             5                |
      0+         1                                   *                +
       |                                             81           *   |
       |                                                              |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
           -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3

                                      ION
```

```
                        Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |                                                              |
       |        1                                                     |
 100000+                                                              +
       |                                           1                  |
       |        2                                   21           1    |
      0+                                         113*8          114   +
       |        1                               1   15          1 3   |
       |                                                          1   |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
        -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3

                                      YAG
```

```
                    Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
 PRICE |                                                               |
200000 +                                                               +
       |                                                               |
       |        1                                                      |
100000 +                                                               +
       |                                 1                             |
       |        2                           32                         |
     0 +                              124*7                            +
       |        1                  1        35                  2 2    |
       |                                                          1    |
-100000+                                                               +
       |                                                               |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3  0.4  0.5

                                   DIODE
```

```
       ----+------+------+------+------+------+------+------+------+----
 PRICE |                                                                |
400000 +                                                             1  +
       |                                                                |
       |                                 1             1                |
       |                                                                |
       |      1                                                         |
200000 +                   1   1     1                                  +
       |     1      1 1    2   1  1                                     |
       |     2   1  1   2                                               |
       |     51  11                                                     |
       |     951                                                        |
     0 +     81                                                         +
       ----+------+------+------+------+------+------+------+------+----
           0    1000   2000   3000   4000   5000   6000   7000   8000

                                   POWER
```

```
           --+------+------+------+------+------+------+------+------+--
  RESIDUAL|                                                            |
   100000+                                                             +
R         |                                  1                         |
e         |                                          1                 |
s         |                  1       1                                 |
i         |      2     1111    1 1 1 1                                 |
d       0+   15212 1  11   1   11 1  11  1              1              +
u         |  44      1 1     1         1                               |
a         |      11                                                   1|
l         |                                                            |
          |          1                                                 |
 -100000+                                                              +
           --+------+------+------+------+------+------+------+------+--
             0    50000 100000 150000 200000 250000 300000 350000 400000

                                   PRICE
```

```
           --+------+------+------+------+------+------+------+------+--
  RESIDUAL|                                                            |
   100000+                                                             +
R         |   1                                                        |
e         |                              1                             |
s         |         2                                                  |
i         |  5  1    1 1  2                                            |
d       0+   *41 1    1  1   11 1                  1                   +
u         |  82  1                  1                                  |
a         |  11                                                    1   |
l         |                                                            |
          |  1                                                         |
 -100000+                                                              +
           --+------+------+------+------+------+------+------+------+--
             0    1000   2000   3000   4000   5000   6000   7000   8000

                                   POWER
```

Appendix C. SAS Output for Model 2

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	357771090943	71554218189	94.226	0.0001
Error	46	34931872065	759388523.15		
C Total	51	392702963008			

Root MSE	27557.00497	R-square	0.9110
Dep Mean	93941.92308	Adj R-sq	0.9014
C.V.	29.33409		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	110815	13862.774556	7.994	0.0001
SQRTPOW	1	3976.462317	202.74155063	19.613	0.0001
CO2	1	-125566	15735.768089	-7.980	0.0001
ION	1	-95839	16339.248370	-5.866	0.0001
YAG	1	-76088	16088.265562	-4.729	0.0001
DIODE	1	-106480	18815.002797	-5.659	0.0001

Variable	DF	Standardized Estimate	Variance Inflation
INTERCEP	1	0.00000000	0.00000000
SQRTPOW	1	0.99719200	1.33674788
CO2	1	-0.70897232	4.08215517
ION	1	-0.43464236	2.83952742
YAG	1	-0.36889476	3.14624340
DIODE	1	-0.36121596	2.10673089

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	7029.1	7861.636	-2029.1	26411.80	-0.077
2	7600.0	10398.5	7752.094	-2798.5	26444.16	-0.106
3	10000.0	13367.0	7657.481	-3367.0	26471.71	-0.127
4	11300.0	20006.4	8716.173	-8706.4	26142.24	-0.333
5	12000.0	20600.1	8715.597	-8600.1	26142.43	-0.329
6	16000.0	22929.5	8714.349	-6929.5	26142.85	-0.265
7	5000.0	5131.5	7924.305	-131.5	26393.07	-0.005
8	10780.0	13367.0	7657.481	-2587.0	26471.71	0.098
9	17000.0	25013.8	7304.741	-8013.8	26571.21	-0.302
10	22000.0	29707.4	7171.590	-7707.4	26607.46	-0.290
11	35000.0	41484.8	6862.890	-6484.8	26688.75	-0.243
12	43000.0	46852.2	6735.282	-3852.2	26721.24	-0.144
13	85000.0	82652.2	6133.627	2347.8	26865.72	0.087
14	24500.0	25497.3	8714.850	-997.3	26142.68	-0.038
15	30100.0	27551.2	8716.666	2548.8	26142.08	0.097
16	30100.0	27551.2	8716.666	2548.8	26142.08	0.097
17	20850.0	21864.0	8714.719	-1014.0	26142.73	-0.039
18	20850.0	21864.0	8714.719	-1014.0	26142.73	-0.039
19	34950.0	23868.2	8714.304	11081.8	26142.87	0.424
20	34950.0	23868.2	8714.304	11081.8	26142.87	0.424
21	23000.0	25013.8	7304.741	-2013.8	26571.21	-0.076
22	45000.0	41484.8	6862.890	3515.2	26688.75	0.132
23	115000	110996	6018.092	4004.1	26891.84	0.149
24	130000	139257	6243.078	-9256.9	26840.50	-0.345
25	140000	163082	6672.893	-23082.0	26736.88	-0.863
26	155000	184072	7201.652	-29072.3	26599.34	-1.093
27	175000	203049	7776.109	-28049.0	26437.11	-1.061
28	190000	220500	8368.550	-30499.8	26255.59	-1.162
29	75000.0	44467.7	8658.108	30532.3	26161.53	1.167
30	85000.0	56507.4	8434.678	28492.6	26234.42	1.086
31	95000.0	62845.2	8332.837	32154.8	26266.94	1.224
32	49000.0	69164.6	8242.661	-20164.6	26295.38	-0.767
33	65000.0	80066.0	8114.859	-15066.0	26335.10	-0.572
34	55000.0	26115.6	12932.28	28884.4	24334.02	1.187
35	75000.0	35137.2	12800.37	39862.8	24403.67	1.633
36	80000.0	113236	12334.91	-33235.5	24642.21	-1.349
37	133000	158343	12642.77	-25343.3	24485.69	-1.035
38	172000	182168	12968.06	-10168.4	24314.98	-0.418
39	155000	178101	8649.447	-23100.8	26164.40	-0.883
40	183000	212560	9477.954	-29560.2	25875.80	-1.142

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	241350	10349.55	-31350.4	25539.68	-1.228
42	43000.0	62845.2	8332.837	-19845.2	26266.94	-0.756
43	105000	123644	7979.081	-18643.8	26376.56	-0.707
44	173000	160474	8337.699	12525.8	26265.40	0.477
45	300000	236743	8964.260	63257.3	26058.21	2.428
46	330000	293265	11268.51	36735.4	25147.75	1.461
47	390000	340915	13386.05	49085.2	24087.39	2.038
48	62000.0	121336	13814.01	-59336.0	23844.53	-2.488
49	122000	141617	13778.57	-19616.8	23865.03	-0.822
50	144000	138933	13778.81	5066.9	23864.89	0.212
51	235000	161114	13817.57	73886.0	23842.47	3.099
52	100000	45974.5	8628.071	54025.5	26171.45	2.064

<table>
<tr><th>Obs</th><th>-2-1-0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>1</td><td>| | |</td><td>0.000</td><td>-0.0760</td><td>0.0814</td><td>1.2411</td><td>-0.0226</td></tr>
<tr><td>2</td><td>| | |</td><td>0.000</td><td>-0.1047</td><td>0.0791</td><td>1.2372</td><td>-0.0307</td></tr>
<tr><td>3</td><td>| | |</td><td>0.000</td><td>-0.1258</td><td>0.0772</td><td>1.2338</td><td>-0.0364</td></tr>
<tr><td>4</td><td>| | |</td><td>0.002</td><td>-0.3298</td><td>0.1000</td><td>1.2496</td><td>-0.1100</td></tr>
<tr><td>5</td><td>| | |</td><td>0.002</td><td>-0.3258</td><td>0.1000</td><td>1.2500</td><td>-0.1086</td></tr>
<tr><td>6</td><td>| | |</td><td>0.001</td><td>-0.2624</td><td>0.1000</td><td>1.2562</td><td>-0.0875</td></tr>
<tr><td>7</td><td>| | |</td><td>0.000</td><td>-0.0049</td><td>0.0827</td><td>1.2438</td><td>-0.0015</td></tr>
<tr><td>8</td><td>| | |</td><td>0.000</td><td>-0.0967</td><td>0.0772</td><td>1.2349</td><td>-0.0280</td></tr>
<tr><td>9</td><td>| | |</td><td>0.001</td><td>-0.2986</td><td>0.0703</td><td>1.2127</td><td>-0.0821</td></tr>
<tr><td>10</td><td>| | |</td><td>0.001</td><td>-0.2868</td><td>0.0677</td><td>1.2105</td><td>-0.0773</td></tr>
<tr><td>11</td><td>| | |</td><td>0.001</td><td>-0.2405</td><td>0.0620</td><td>1.2071</td><td>-0.0618</td></tr>
<tr><td>12</td><td>| | |</td><td>0.000</td><td>-0.1426</td><td>0.0597</td><td>1.2102</td><td>-0.0359</td></tr>
<tr><td>13</td><td>| | |</td><td>0.000</td><td>0.0864</td><td>0.0495</td><td>1.1992</td><td>0.0197</td></tr>
<tr><td>14</td><td>| | |</td><td>0.000</td><td>-0.0377</td><td>0.1000</td><td>1.2675</td><td>-0.0126</td></tr>
<tr><td>15</td><td>| | |</td><td>0.000</td><td>0.0964</td><td>0.1001</td><td>1.2662</td><td>0.0322</td></tr>
<tr><td>16</td><td>| | |</td><td>0.000</td><td>0.0964</td><td>0.1001</td><td>1.2662</td><td>0.0322</td></tr>
<tr><td>17</td><td>| | |</td><td>0.000</td><td>-0.0384</td><td>0.1000</td><td>1.2675</td><td>-0.0128</td></tr>
<tr><td>18</td><td>| | |</td><td>0.000</td><td>-0.0384</td><td>0.1000</td><td>1.2675</td><td>-0.0128</td></tr>
<tr><td>19</td><td>| | |</td><td>0.003</td><td>0.4201</td><td>0.1000</td><td>1.2383</td><td>0.1400</td></tr>
<tr><td>20</td><td>| | |</td><td>0.003</td><td>0.4201</td><td>0.1000</td><td>1.2383</td><td>0.1400</td></tr>
<tr><td>21</td><td>| | |</td><td>0.000</td><td>-0.0750</td><td>0.0703</td><td>1.2263</td><td>-0.0206</td></tr>
<tr><td>22</td><td>| | |</td><td>0.000</td><td>0.1303</td><td>0.0620</td><td>1.2137</td><td>0.0335</td></tr>
<tr><td>23</td><td>| | |</td><td>0.000</td><td>0.1473</td><td>0.0477</td><td>1.1946</td><td>0.0330</td></tr>
<tr><td>24</td><td>| | |</td><td>0.001</td><td>-0.3416</td><td>0.0513</td><td>1.1842</td><td>-0.0794</td></tr>
<tr><td>25</td><td>| *| |</td><td>0.008</td><td>-0.8609</td><td>0.0586</td><td>1.0989</td><td>-0.2149</td></tr>
</table>

<table>
<tr><th>Obs</th><th>-2-1-0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>26</td><td>| **| |</td><td>0.015</td><td>-1.0953</td><td>0.0683</td><td>1.0458</td><td>-0.2966</td></tr>
<tr><td>27</td><td>| **| |</td><td>0.016</td><td>-1.0625</td><td>0.0796</td><td>1.0684</td><td>-0.3125</td></tr>
<tr><td>28</td><td>| **| |</td><td>0.023</td><td>-1.1662</td><td>0.0922</td><td>1.0513</td><td>-0.3717</td></tr>
<tr><td>29</td><td>| |** |</td><td>0.025</td><td>1.1718</td><td>0.0987</td><td>1.0570</td><td>0.3878</td></tr>
<tr><td>30</td><td>| |** |</td><td>0.020</td><td>1.0883</td><td>0.0937</td><td>1.0772</td><td>0.3499</td></tr>
<tr><td>31</td><td>| |** |</td><td>0.025</td><td>1.2310</td><td>0.0914</td><td>1.0295</td><td>0.3905</td></tr>
<tr><td>32</td><td>| *| |</td><td>0.010</td><td>-0.7634</td><td>0.0895</td><td>1.1600</td><td>-0.2393</td></tr>
<tr><td>33</td><td>| *| |</td><td>0.005</td><td>-0.5679</td><td>0.0867</td><td>1.1969</td><td>-0.1750</td></tr>
<tr><td>34</td><td>| |** |</td><td>0.066</td><td>1.1924</td><td>0.2202</td><td>1.2141</td><td>0.6337</td></tr>
<tr><td>35</td><td>| |*** |</td><td>0.122</td><td>1.6646</td><td>0.2158</td><td>1.0165</td><td>0.8731</td></tr>
<tr><td>36</td><td>| **| |</td><td>0.076</td><td>-1.3612</td><td>0.2004</td><td>1.1201</td><td>-0.6813</td></tr>
<tr><td>37</td><td>| **| |</td><td>0.048</td><td>-1.0358</td><td>0.2105</td><td>1.2546</td><td>-0.5348</td></tr>
<tr><td>38</td><td>| | |</td><td>0.008</td><td>-0.4144</td><td>0.2215</td><td>1.4324</td><td>-0.2210</td></tr>
<tr><td>39</td><td>| *| |</td><td>0.014</td><td>-0.8808</td><td>0.0985</td><td>1.1423</td><td>-0.2912</td></tr>
<tr><td>40</td><td>| **| |</td><td>0.029</td><td>-1.1463</td><td>0.1183</td><td>1.0888</td><td>-0.4199</td></tr>
<tr><td>41</td><td>| **| |</td><td>0.041</td><td>-1.2345</td><td>0.1411</td><td>1.0877</td><td>-0.5003</td></tr>
<tr><td>42</td><td>| *| |</td><td>0.010</td><td>-0.7519</td><td>0.0914</td><td>1.1651</td><td>-0.2385</td></tr>
<tr><td>43</td><td>| *| |</td><td>0.008</td><td>-0.7029</td><td>0.0838</td><td>1.1664</td><td>-0.2126</td></tr>
<tr><td>44</td><td>| | |</td><td>0.004</td><td>0.4729</td><td>0.0915</td><td>1.2191</td><td>0.1501</td></tr>
<tr><td>45</td><td>| |**** |</td><td>0.116</td><td>2.5714</td><td>0.1058</td><td>0.5606</td><td>0.8846</td></tr>
<tr><td>46</td><td>| |** |</td><td>0.071</td><td>1.4795</td><td>0.1672</td><td>1.0303</td><td>0.6630</td></tr>
<tr><td>47</td><td>| |**** |</td><td>0.214</td><td>2.1132</td><td>0.2360</td><td>0.8465</td><td>1.1743</td></tr>
<tr><td>48</td><td>|****| |</td><td>0.346</td><td>-2.6458</td><td>0.2513</td><td>0.6400</td><td>-1.5328</td></tr>
<tr><td>49</td><td>| *| |</td><td>0.038</td><td>-0.8190</td><td>0.2500</td><td>1.3920</td><td>-0.4729</td></tr>
<tr><td>50</td><td>| | |</td><td>0.003</td><td>0.2101</td><td>0.2500</td><td>1.5124</td><td>0.1213</td></tr>
<tr><td>51</td><td>| |*****|</td><td>0.538</td><td>3.4458</td><td>0.2514</td><td>0.3740</td><td>1.9969</td></tr>
<tr><td>52</td><td>| |**** |</td><td>0.077</td><td>2.1434</td><td>0.0980</td><td>0.7059</td><td>0.7066</td></tr>
</table>

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas	DIODE Dfbetas
1	-0.0016	0.0146	-0.0109	0.0010	-0.0022	-0.0027
2	-0.0021	0.0194	-0.0148	0.0013	-0.0029	-0.0036
3	-0.0025	0.0225	-0.0176	0.0015	-0.0033	-0.0042
4	-0.0003	0.0023	-0.0007	-0.0585	-0.0003	-0.0004
5	-0.0002	0.0019	-0.0006	-0.0578	-0.0003	-0.0004
6	-0.0000	0.0003	-0.0001	-0.0466	-0.0000	-0.0001
7	-0.0001	0.0010	-0.0007	0.0001	-0.0001	-0.0002
8	-0.0019	0.0173	-0.0135	0.0012	-0.0026	-0.0032
9	-0.0051	0.0466	-0.0396	0.0031	-0.0069	-0.0087
10	-0.0046	0.0421	-0.0372	0.0028	-0.0063	-0.0078
11	-0.0033	0.0298	-0.0295	0.0020	-0.0044	-0.0056
12	-0.0018	0.0162	-0.0170	0.0011	-0.0024	-0.0030
13	0.0004	-0.0039	0.0085	-0.0003	0.0006	0.0007
14	0.0000	-0.0001	0.0000	-0.0067	0.0000	0.0000
15	-0.0001	0.0008	-0.0002	0.0172	-0.0001	-0.0001
16	-0.0001	0.0008	-0.0002	0.0172	-0.0001	-0.0001
17	-0.0000	0.0001	-0.0000	-0.0068	-0.0000	-0.0000
18	-0.0000	0.0001	-0.0000	-0.0068	-0.0000	-0.0000
19	-0.0000	0.0003	-0.0001	0.0747	-0.0000	-0.0000
20	-0.0000	0.0003	-0.0001	0.0747	-0.0000	-0.0000
21	-0.0013	0.0117	-0.0099	0.0008	-0.0017	-0.0022
22	0.0018	-0.0161	0.0160	-0.0011	0.0024	0.0030
23	-0.0001	0.0013	0.0122	0.0001	-0.0002	-0.0002
24	0.0024	-0.0213	-0.0229	-0.0014	0.0032	0.0040
25	0.0103	-0.0931	-0.0465	-0.0062	0.0138	0.0173
26	0.0180	-0.1632	-0.0464	-0.0109	0.0242	0.0304
27	0.0218	-0.1981	-0.0338	-0.0132	0.0294	0.0369
28	0.0285	-0.2585	-0.0257	-0.0172	0.0384	0.0481
29	0.0169	-0.1531	0.0452	-0.0102	0.1989	0.0285
30	0.0128	-0.1163	0.0344	-0.0077	0.1804	0.0217
31	0.0128	-0.1163	0.0343	-0.0077	0.2016	0.0217
32	-0.0069	0.0627	-0.0185	0.0042	-0.1235	-0.0117
33	-0.0038	0.0346	-0.0102	0.0023	-0.0899	-0.0064
34	0.0211	-0.1921	0.0567	-0.0128	0.0285	0.4313
35	0.0260	-0.2360	0.0697	-0.0157	0.0350	0.5946
36	0.0032	-0.0288	0.0085	-0.0019	0.0043	-0.4405
37	0.0131	-0.1194	0.0353	-0.0080	0.0177	-0.3193
38	0.0076	-0.0688	0.0203	-0.0046	0.0102	-0.1248
39	0.0126	-0.1143	0.0338	-0.0076	-0.1154	0.0213
40	0.0251	-0.2283	0.0674	-0.0152	-0.1404	0.0425

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas	DIODE Dfbetas
41	0.0352	-0.3200	0.0945	-0.0213	-0.1426	0.0596
42	-0.0078	0.0710	-0.0210	0.0047	-0.1231	-0.0132
43	0.0018	-0.0165	0.0049	-0.0011	-0.1024	0.0031
44	-0.0049	0.0450	-0.0133	0.0030	0.0641	-0.0084
45	-0.0722	0.6560	0.0330	0.0437	-0.0974	-0.1222
46	-0.0617	0.5607	-0.0304	0.0374	-0.0832	-0.1044
47	-0.1155	1.0492	-0.1083	0.0699	-0.1557	-0.1954
48	-1.5317	0.1098	1.3062	1.2966	1.2931	1.0991
49	-0.4698	-0.0015	0.4145	0.3987	0.4052	0.3466
50	0.1207	-0.0008	-0.1060	-0.1023	-0.1038	-0.0887
51	1.9627	0.1501	-1.7879	-1.6692	-1.7277	-1.4862
52	0.0301	-0.2736	0.0808	-0.0182	0.3628	0.0510

```
                    Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE |                                                                |
 200000+                                                                 +
        |                                                           1    |
        |                                                                |
 100000+                                                            2    +
        |       1   1                                                1   |
        |     11    41                                                   |
     0+        12*48                                                     +
        |       14322                                                    |
        |                                                                |
-100000+                                                                 +
        |                                                                |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.1  0.0  0.1  0.2  0.3  0.4  0.5  0.6  0.7  0.8  0.9  1.0

                                INTERCEP
```

Partial Regression Residual Plot

```
       -+------+------+------+------+------+------+------+------+-----+
  PRICE|                                                              |
 400000+                                                              +
       |                                                              |
       |                                                           1  |
 200000+                                             1         1      +
       |                                                              |
       |                          1    11 1 2  11 1  1                |
      0+          2 11    1  2832   1   1                             +
       |     12 311211 11  1                                          |
       |    11                                                        |
-200000+                                                              +
       |                                                              |
       -+------+------+------+------+------+------+------+------+-----+
       -30    -20    -10     0     10     20     30     40     50    60
```

SQRTPOW

Partial Regression Residual Plot

```
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |      1                                                       |
       |                                                              |
 100000+       1                                                      +
       |       1                                 1    1               |
       |        1                                  2   141            |
      0+                                       1 11  *      1         +
       |                                        1 1 11 41111 1 335    |
       |                                                              |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3
```

CO2

```
                       Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |                                                              |
       |          1                                                   |
 100000+                                                              +
       |         1                                  1 1               |
       |         1                                  5  2              |
      0+         1                                  *42            2  +
       |                                            138            8  |
       |                                                              |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3

                                       ION
```

```
                       Partial Regression Residual Plot
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |                                                              |
       |        1                                                     |
 100000+                                                              +
       |        1                              1  1                   |
       |        1                                1     2           1  |
      0+         1                                 11*2*        1 12  +
       |                                          1221        11113   |
       |                                                              |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3

                                       YAG
```

Partial Regression Residual Plot

```
       ----+----+----+----+----+----+----+----+----+----+----+----+----
  PRICE|                                                              |
 200000+                                                              +
       |                                                              |
       |        1                                                     |
 100000+                                                              +
       |        1                    1  1    1                        |
       |        1                     1     21                        |
      0+         1                        2*192                   1   +
       |                                33113                     1   |
       |                                                      111     |
-100000+                                                              +
       |                                                              |
       ----+----+----+----+----+----+----+----+----+----+----+----+----
         -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3  0.4  0.5

                                 DIODE
```

```
      +------+------+------+------+------+------+------+------+------+------+-
 PRICE|                                                               |
400000+                                                            1  +
      |                                                               |
      |                                             1         1       |
      |                                                               |
      |          1                                                    |
200000+                                1    1    1                    +
      |      1                1  1     2   1  1                       |
      |   1  1          1     1    2                                  |
      |   2 12  1        1 1                                          |
      |  25 11111 21                                                  |
     0+  3  32 1                                                      +
      +------+------+------+------+------+------+------+------+------+------+-
      0     10     20     30     40     50     60     70     80     90

                                 SQRTPOW
```

```
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
  RESIDUAL|                                                           |
    100000+                                                           +
R         |            1                                              |
e         |      1                                   1                |
s         |     1  11                                        1       1|
i         |     2 2               1                                   |
d        0+     5324 2121     1   1   1                               +
u         |        12 1     1        11   3  1  1                     |
a         |                    1              1   1                   |
l         |      1                                                    |
          |                                                           |
   -100000+                                                           +
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
             0    10    20    30    40    50    60    70    80    90

                                SQRTPOW
```

Appendix D. SAS Output for Model 3

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	354231179753	70846235951	216.293	0.0001
Error	41	13429430128	327547076.29		
C Total	46	367660609881	.		

Root MSE	18098.26169	R-square	0.9635
Dep Mean	89829.36170	Adj R-sq	0.9590
C.V.	20.14738		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	B	30596	10137.675177	3.018	0.0044
SQRTPOW	1	2905.630646	248.02492517	11.715	0.0001
WEIGHT	1	18.660509	3.57530562	5.219	0.0001
CO2	B	-34047	9475.9262404	-3.593	0.0009
ION	B	-14286	11448.165730	-1.248	0.2191
YAG	B	6060.906812	10465.090162	0.579	0.5657
DIODE	0	0	.	.	.

Variable	DF	Standardized Estimate	Variance Inflation
INTERCEP	B	0.00000000	0.00000000
SQRTPOW	1	0.73636328	4.43473286
WEIGHT	1	0.30471908	3.82606239
CO2	B	-0.19138411	3.18466973
ION	B	-0.06610738	3.14993763
YAG	B	0.02901434	2.81714726
DIODE	0	.	.

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	12799.6	5317.873	-7799.6	17299.34	-0.451
2	7600.0	15336.3	212.372	-7736.3	17331.42	-0.446
3	10000.0	17580.0	5124.278	-7580.0	17357.67	-0.437
4	11300.0	20477.8	5725.193	-9177.8	17168.85	-0.535
5	12000.0	20911.6	5724.385	-8911.6	17169.11	-0.519
6	16000.0	22613.7	5723.527	-6613.7	17169.40	-0.385
7	5000.0	11413.0	5381.520	-6413.0	17279.65	-0.371
8	10780.0	17915.9	5135.768	-7135.9	17354.28	-0.411
9	17000.0	26165.0	4820.780	-9165.0	17444.40	-0.525
10	22000.0	29706.6	4723.760	-7706.6	17470.92	-0.441
11	35000.0	38947.0	4537.937	-3947.0	17520.11	-0.225
12	43000.0	43130.2	4481.500	-130.2	17534.63	-0.007
13	85000.0	70521.1	4642.332	14478.9	17492.74	0.828
14	24500.0	25434.3	5723.564	-934.3	17169.39	-0.054
15	30100.0	26636.5	5727.546	3463.5	17168.06	0.202
16	30100.0	26636.5	5727.546	3463.5	17168.06	0.202
17	20850.0	22480.8	5724.709	-1630.8	17169.01	-0.095
18	20850.0	22480.8	5724.709	-1630.8	17169.01	-0.095
19	34950.0	23964.0	5723.211	10986.0	17169.51	0.640
20	34950.0	23964.0	5723.211	10986.0	17169.51	0.640
21	23000.0	26918.9	4831.547	-3918.9	17441.42	-0.225
22	45000.0	39426.6	4530.096	5573.4	17522.14	0.318
23	115000	108960	3972.071	6040.4	17657.00	0.342
24	130000	129610	4511.299	389.9	17526.99	0.022
25	140000	147019	5388.517	-7019.2	17277.47	-0.406
26	155000	162357	6351.914	-7357.0	16946.98	-0.434
27	175000	178276	7040.771	-3276.1	16672.57	-0.196
28	190000	191028	7965.107	-1027.6	16251.28	-0.063
29	75000.0	44800.7	6082.645	30199.3	17045.48	1.772
30	85000.0	53971.4	5854.675	31028.6	17125.12	1.812
31	95000.0	58602.5	5763.240	36397.5	17156.11	2.122
32	49000.0	63220.1	5697.976	-14220.1	17177.90	-0.828
33	65000.0	71652.4	5637.771	-6652.4	17197.75	-0.387
34	55000.0	46790.9	9405.403	8209.1	15462.39	0.531
35	75000.0	53383.0	9131.626	21617.0	15625.64	1.383
36	80000.0	110499	8118.684	-30498.6	16175.11	-1.886
37	133000	143459	8797.941	-10459.2	15815.92	-0.661
38	172000	160868	9479.574	11131.6	15417.03	0.722
39	155000	193671	6999.212	-38670.5	16690.06	-2.317
40	183000	218850	6662.456	-35850.3	16827.32	-2.130

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	239888	6901.370	-29887.5	16730.76	-1.786
42	43000.0	58695.8	5763.804	-15695.8	17155.92	-0.915
43	105000	103682	6177.848	1318.4	17011.21	0.078
44	173000	130967	7377.935	42032.9	16526.14	2.543
45	300000	282950	10560.08	17050.1	14698.02	1.160
46	330000	324251	9431.985	5749.1	15446.19	0.372
47	390000	359069	9447.791	30930.8	15436.53	2.004
48	62000.0	.	.	.	.	.
49	122000	.	.	.	.	.
50	144000	.	.	.	.	.
51	235000	.	.	.	.	.
52	100000	.	.	.	.	.

Obs	-2-1-0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
1	| | |	0.003	-0.4464	0.0863	1.2320	-0.1372
2	| | |	0.003	-0.4420	0.0829	1.2282	-0.1329
3	| | |	0.003	-0.4323	0.0802	1.2260	-0.1276
4	| *| |	0.005	-0.5299	0.1001	1.2357	-0.1767
5	| *| |	0.005	-0.5144	0.1000	1.2386	-0.1715
6	| | |	0.003	-0.3812	0.1000	1.2608	-0.1271
7	| | |	0.002	-0.3672	0.0884	1.2467	-0.1144
8	| | |	0.002	-0.4070	0.0805	1.2304	-0.1204
9	| *| |	0.004	-0.5207	0.0710	1.1987	-0.1439
10	| | |	0.002	-0.4367	0.0681	1.2095	-0.1181
11	| | |	0.001	-0.2227	0.0629	1.2283	-0.0577
12	| | |	0.000	-0.0073	0.0613	1.2354	-0.0019
13	| |* |	0.008	0.8245	0.0658	1.1220	0.2188
14	| | |	0.000	-0.0537	0.1000	1.2880	-0.0179
15	| | |	0.001	0.1994	0.1002	1.2811	0.0665
16	| | |	0.001	0.1994	0.1002	1.2811	0.0665
17	| | |	0.000	-0.0938	0.1001	1.2869	-0.0313
18	| | |	0.000	-0.0938	0.1001	1.2869	-0.0313
19	| |* |	0.008	0.6352	0.1000	1.2132	0.2117
20	| |* |	0.008	0.6352	0.1000	1.2132	0.2117
21	| | |	0.001	-0.2221	0.0713	1.2395	-0.0615
22	| | |	0.001	0.3146	0.0627	1.2190	0.0813
23	| | |	0.001	0.3384	0.0482	1.1977	0.0761
24	| | |	0.000	0.0220	0.0621	1.2364	0.0057
25	| | |	0.003	-0.4021	0.0886	1.2421	-0.1254

<table>
<tr><th>Obs</th><th>-2-1-0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>26</td><td>| | |</td><td>0.004</td><td>-0.4298</td><td>0.1232</td><td>1.2866</td><td>-0.1611</td></tr>
<tr><td>27</td><td>| | |</td><td>0.001</td><td>-0.1942</td><td>0.1513</td><td>1.3588</td><td>-0.0820</td></tr>
<tr><td>28</td><td>| | |</td><td>0.000</td><td>-0.0625</td><td>0.1937</td><td>1.4374</td><td>-0.0306</td></tr>
<tr><td>29</td><td>| |*** |</td><td>0.067</td><td>1.8210</td><td>0.1130</td><td>0.8107</td><td>0.6498</td></tr>
<tr><td>30</td><td>| |*** |</td><td>0.064</td><td>1.8659</td><td>0.1046</td><td>0.7850</td><td>0.6379</td></tr>
<tr><td>31</td><td>| |**** |</td><td>0.085</td><td>2.2210</td><td>0.1014</td><td>0.6423</td><td>0.7461</td></tr>
<tr><td>32</td><td>| *| |</td><td>0.013</td><td>-0.8246</td><td>0.0991</td><td>1.1635</td><td>-0.2735</td></tr>
<tr><td>33</td><td>| | |</td><td>0.003</td><td>-0.3828</td><td>0.0970</td><td>1.2565</td><td>-0.1255</td></tr>
<tr><td>34</td><td>| |* |</td><td>0.017</td><td>0.5262</td><td>0.2701</td><td>1.5244</td><td>0.3201</td></tr>
<tr><td>35</td><td>| |** |</td><td>0.109</td><td>1.3995</td><td>0.2546</td><td>1.1678</td><td>0.8179</td></tr>
<tr><td>36</td><td>| ***| |</td><td>0.149</td><td>-1.9488</td><td>0.2012</td><td>0.8425</td><td>-0.9781</td></tr>
<tr><td>37</td><td>| *| |</td><td>0.023</td><td>-0.6567</td><td>0.2363</td><td>1.4239</td><td>-0.3653</td></tr>
<tr><td>38</td><td>| |* |</td><td>0.033</td><td>0.7178</td><td>0.2743</td><td>1.4800</td><td>0.4413</td></tr>
<tr><td>39</td><td>|****| |</td><td>0.157</td><td>-2.4549</td><td>0.1496</td><td>0.5875</td><td>-1.0295</td></tr>
<tr><td>40</td><td>|****| |</td><td>0.119</td><td>-2.2315</td><td>0.1355</td><td>0.6635</td><td>-0.8835</td></tr>
<tr><td>41</td><td>| ***| |</td><td>0.090</td><td>-1.8374</td><td>0.1454</td><td>0.8345</td><td>-0.7579</td></tr>
<tr><td>42</td><td>| *| |</td><td>0.016</td><td>-0.9130</td><td>0.1014</td><td>1.1404</td><td>-0.3067</td></tr>
<tr><td>43</td><td>| | |</td><td>0.000</td><td>0.0766</td><td>0.1165</td><td>1.3115</td><td>0.0278</td></tr>
<tr><td>44</td><td>| |*****|</td><td>0.215</td><td>2.7374</td><td>0.1662</td><td>0.4964</td><td>1.2221</td></tr>
<tr><td>45</td><td>| |** |</td><td>0.116</td><td>1.1651</td><td>0.3405</td><td>1.4393</td><td>0.8371</td></tr>
<tr><td>46</td><td>| | |</td><td>0.009</td><td>0.3683</td><td>0.2716</td><td>1.5601</td><td>0.2249</td></tr>
<tr><td>47</td><td>| |**** |</td><td>0.251</td><td>2.0838</td><td>0.2725</td><td>0.8590</td><td>1.2754</td></tr>
<tr><td>48</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>49</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>50</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>51</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>52</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
</table>

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas
1	-0.0440	0.0727	-0.0306	-0.0402	0.0360	0.0154
2	-0.0404	0.0668	-0.0261	-0.0407	0.0331	0.0136
3	-0.0369	0.0609	-0.0220	-0.0406	0.0302	0.0119
4	-0.0027	0.0044	-0.0029	0.0006	-0.0861	0.0012
5	-0.0019	0.0032	-0.0019	0.0003	-0.0841	0.0008
6	0.0005	-0.0009	0.0013	-0.0004	-0.0640	-0.0005
7	-0.0377	0.0624	-0.0273	-0.0326	0.0309	0.0135
8	-0.0355	0.0586	-0.0222	-0.0378	0.0290	0.0117
9	-0.0313	0.0517	-0.0086	-0.0531	0.0255	0.0073
10	-0.0221	0.0364	-0.0016	-0.0459	0.0179	0.0039
11	-0.0064	0.0105	0.0055	-0.0248	0.0051	-0.0005
12	-0.0001	0.0002	0.0003	-0.0008	0.0001	-0.0001
13	-0.0424	0.0708	-0.1086	0.1133	0.0360	0.0368
14	0.0001	-0.0001	0.0000	0.0000	-0.0090	-0.0000
15	-0.0016	0.0026	-0.0021	0.0005	0.0345	0.0008
16	-0.0016	0.0026	-0.0021	0.0005	0.0345	0.0008
17	-0.0004	0.0007	-0.0007	0.0002	-0.0153	0.0002
18	-0.0004	0.0007	-0.0007	0.0002	-0.0153	0.0002
19	0.0002	-0.0004	0.0007	-0.0002	0.1057	-0.0002
20	0.0002	-0.0004	0.0007	-0.0002	0.1057	-0.0002
21	-0.0143	0.0236	-0.0055	-0.0221	0.0116	0.0038
22	0.0099	-0.0162	-0.0061	0.0345	-0.0079	0.0002
23	-0.0048	0.0080	-0.0076	0.0336	0.0040	0.0028
24	-0.0016	0.0027	-0.0024	0.0027	0.0014	0.0009
25	0.0514	-0.0853	0.0727	-0.0566	-0.0427	-0.0282
26	0.0760	-0.1261	0.1071	-0.0686	-0.0631	-0.0417
27	0.0409	-0.0679	0.0562	-0.0330	-0.0339	-0.0220
28	0.0160	-0.0266	0.0221	-0.0118	-0.0133	-0.0086
29	0.1441	-0.2385	0.1138	-0.0131	-0.1183	0.2498
30	0.1016	-0.1679	0.0551	0.0014	-0.0830	0.2788
31	0.0899	-0.1483	0.0224	0.0124	-0.0731	0.3481
32	-0.0220	0.0361	0.0076	-0.0086	0.0176	-0.1353
33	-0.0021	0.0033	0.0142	-0.0065	0.0014	-0.0670
34	0.3181	-0.1630	0.1370	-0.2694	-0.2763	-0.2664
35	0.8068	-0.3787	0.3181	-0.6983	-0.7019	-0.6847
36	-0.7325	-0.0765	0.0642	0.8170	0.6511	0.7291
37	-0.1821	-0.1432	0.1203	0.2571	0.1660	0.2130
38	0.1625	0.2297	-0.1930	-0.2738	-0.1515	-0.2161
39	-0.1882	0.3155	-0.5698	0.1954	0.1612	-0.2307
40	0.0020	-0.0006	-0.2742	0.1165	0.0028	-0.3014

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	CO2 Dfbetas	ION Dfbetas	YAG Dfbetas
41	0.1204	-0.1977	-0.0614	0.0553	-0.0963	-0.3143
42	-0.0374	0.0618	-0.0102	-0.0048	0.0304	-0.1427
43	-0.0069	0.0114	-0.0130	0.0038	0.0058	0.0175
44	-0.4759	0.7915	-0.7865	0.2171	0.3973	0.7643
45	0.2347	-0.3933	0.6933	-0.1058	-0.2008	-0.2294
46	0.0200	-0.0342	0.1379	-0.0143	-0.0183	-0.0414
47	-0.1285	0.2074	0.4435	0.0032	0.0968	-0.1031
48	.	.	.	.	.	.
49	.	.	.	.	.	.
50	.	.	.	.	.	.
51	.	.	.	.	.	.
52	.	.	.	.	.	.

```
                         Partial Regression Residual Plot
      +----------+----------+----------+----------+----------+--------+
PRICE |                                                               |
50000+                                                              1 +
      |                       1                                      1|
      |        1       1       1 1       1                            |
      |                                    1                          |
      |                1  12 1 1                                      |
    0+              11    2311  1131         1                        +
      |  1  1    1         4 1111                   1                 |
      |       1                                                       |
      |               1          1                                    |
      |                    1                                          |
-50000+                                                               +
      +----------+----------+----------+----------+----------+--------+
    -0.50      -0.25       0.00       0.25       0.50       0.75    1.00

                                  INTERCEP
```

Partial Regression Residual Plot

```
       -+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
  PRICE|                                                              |
 100000+                                                 1            +
       |                                                           1  |
       |                                                  2   1       |
       |                                 2    1     1                 |
       |                   11     2 1        1     1                  |
      0+               1      11 322       1                          +
       |      1          1121 2  2   1                                |
       |  11        212           1                                   |
       |                1                                             |
       |                                                              |
-100000+                                                              +
       -+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
       -20   -15   -10   -5     0     5    10    15    20    25    30

                                SQRTPOW
```

Partial Regression Residual Plot

```
      +------+------+------+------+------+------+------+------+------+-
 PRICE|                                                               |
100000+                                                               +
      |                                                               |
      |                                                              1|
 50000+                                          1                    +
      |                              11 1         1 1      1          |
      |          1                  2                                 |
     0+                    1     123511122                            +
      |             1   11   1   1 121             1                  |
      |       1  1 1     1        1   1     1                         |
-50000+                                                               +
      |                                                               |
      +------+------+------+------+------+------+------+------+------+-
    -2000  -1500  -1000  -500     0     500   1000   1500   2000   2500

                                 WEIGHT
```

```
                       Partial Regression Residual Plot
      +----+----+----+----+----+----+----+----+----+----+----+----+---+
PRICE |                                                               |
 50000+    1                                                          +
      |   1                                          1     1          |
      |              1                             12                 |
      |                                      1                        |
      |            1                             1  2            1    |
     0+                                             5   1    11       +
      |         1                                   311     3322  11  |
      |                                             1        1   11   |
      |                                      1 1 1                    |
      |                                                               |
-50000+                                                               +
      +----+----+----+----+----+----+----+----+----+----+----+----+---+
    -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0.0  0.1  0.2  0.3 0.4

                                    CO2
```

```
                      Partial Regression Residual Plot
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---+
PRICE |                                                               |
 50000+                                                               +
      |                                             1               1 |
      |       1                                  1 1       1          |
      |     1                         1   1                           |
      |                                            1 1  1  1         2|
     0+                                        1  1 11      11      12+
      |                           1            22 211  1        1 6  1|
      |                    1                                          |
      |                                                     1         |
      |                                          1     1              |
-50000+                                                               +
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---+
    -0.8  -0.7  -0.6  -0.5  -0.4  -0.3  -0.2  -0.1  0.0   0.1   0.2 0.3

                                    ION
```

Partial Regression Residual Plot

```
      +----+----+----+----+----+----+----+----+----+----+----+----+---+
 PRICE|                                                               |
 50000+                                                               +
      |                                                     1        1|
      |                                    1              11          |
      |   1                      1                                    |
      |                  1                     31  1                  |
     0+   1                            1      161 1     1 1  1    1   +
      |               1                     3223     1 1    2         |
      |                                                               |
      |           1                                       1  1        |
      |                                               1               |
-50000+                                                               +
      +----+----+----+----+----+----+----+----+----+----+----+----+---+
    -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  .1   0.2  0.3 0.4 0.5
```

YAG

Partial Regression Residual Plot

```
      ----+------+------+------+------+------+------+------+------+----
 PRICE|                                                               |
 50000+                                                               +
      |                        1                       1              |
      |                                        1           2          |
      |                                                1          1   |
      |         1                              1   2   3   1          |
     0+                 1      1       1       1   3   4   1          +
      |                 1      1       1           2   3   2   3  2   |
      |                                                1              |
      |                        1       1                              |
      |                                            1       1          |
-50000+                                                               +
      ----+------+------+------+------+------+------+------+------+----
       -1E-15 -8E-16 -6E-16 -4E-16 -2E-16    0    2E-16  4E-16  6E-16
```

DIODE

```
        +-------+-------+-------+-------+-------+-------+-------+-------+-------+-
PRICE   |                                                                         |
 400000 +                                                                     1   +
        |                                                                         |
        |                                              1          1               |
        |                                                                         |
        |                                                                         |
 200000 +                                    1    1    1                          +
        |                        1  1        2   1  1                             |
        |                  1     1    2                                           |
        |   1 12   1        1 1                                                   |
        |  25 11111 21                                                            |
      0 +  3   32 1                                                               +
        +-------+-------+-------+-------+-------+-------+-------+-------+-------+-
        0      10      20      30      40      50      60      70      80      90

                                      SQRTPOW
```

```
       +----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
PRICE  |                                                                 |
400000 +                                                               1+
       |                                                                 |
       |                                                               2|
       |                                                                 |
       |                                                                 |
200000 +             1                  2                                +
       |1 1        2 1                  1                                |
       |11         2                                                     |
       |241                                                              |
       |2*                                                               |
     0 +81                                                               +
       +----+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
       0   500  1000  1500  2000  2500  3000  3500  4000  4500  5000 5500

                                     WEIGHT
```

```
           ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
   RESIDUAL|                                                           |
      50000+                                                           +
 R         |        1              1                                   |
 e         |     1 1                                                  1|
 s         |         1                                1                |
 i         |     2 1    1      1   1       1                  1        |
 d       0+     23   1 11   1         1         1 1                   +
 u         |     3 23112               1   1  1                        |
 a         |        1                                                  |
 l         |                    1              1                       |
           |                          1    1                           |
     -50000+                                                           +
           ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
              0     10    20    30    40    50    60    70    80    90

                                 SQRTPOW
```

```
           --+----+----+----+----+----+----+----+----+----+----+----+---
   RESIDUAL|                                                           |
      50000+                                                           +
 R         |    2                                                      |
 e         |    2                                                     1|
 s         |   1                                                      1|
 i         |   231        1                                           1|
 d       0+    9         12                                           +
 u         |   *2         2                                            |
 a         |    1                                                      |
 l         |   1                           1                           |
           |                               2                           |
     -50000+                                                           +
           --+----+----+----+----+----+----+----+----+----+----+----+---
             0   500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500

                                 WEIGHT
```

Appendix E. SAS Output for Model 4

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	354121313834	88530328459	274.628	0.0001
Error	42	13539296047	322364191.59		
C Total	46	367660609881			

Root MSE	17954.50338	R-square	0.9632
Dep Mean	89829.36170	Adj R-sq	0.9597
C.V.	19.98734		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	35377	5836.8991240	6.061	0.0001
SQRTPOW	1	2858.551919	232.46435947	12.297	0.0001
WEIGHT	1	19.399298	3.31346555	5.855	0.0001
CO2	1	-38239	6068.6429186	-6.301	0.0001
ION	1	-19005	7978.9057041	-2.382	0.0218

Variable	DF	Standardized Estimate	Variance Inflation
INTERCEP	1	0.00000000	0.00000000
SQRTPOW	1	0.72443229	3.95837102
WEIGHT	1	0.31678322	3.33900874
CO2	1	-0.21494386	1.32718407
ION	1	-0.08794081	1.55468976

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	13145.0	5242.344	-8145.0	17172.13	-0.474
2	7600.0	15644.8	5143.896	-8044.8	17201.88	-0.468
3	10000.0	17856.3	5061.493	-7856.3	17226.30	-0.456
4	11300.0	20500.7	5679.581	-9200.7	17032.51	-0.540
5	12000.0	20927.5	5678.850	-8927.5	17032.76	-0.524
6	16000.0	22602.0	5678.028	-6602.0	17033.03	-0.388
7	5000.0	11780.9	5301.448	-6780.9	17153.97	-0.395
8	10780.0	18205.5	5070.765	-7425.5	17223.57	-0.431
9	17000.0	26306.4	4776.352	-9306.4	17307.53	-0.538
10	22000.0	29796.9	4683.688	-7796.9	17332.84	-0.450
11	35000.0	38922.9	4501.702	-3922.9	17380.99	-0.226
12	43000.0	43053.0	4443.931	-52.9658	17395.85	-0.003
13	85000.0	70068.8	4539.801	14931.2	17371.08	0.860
14	24500.0	25429.5	5678.094	-929.5	17033.01	-0.055
15	30100.0	26595.6	5681.619	3504.4	17031.83	0.206
16	30100.0	26595.6	5681.619	3504.4	17031.83	0.206
17	20850.0	22507.2	5679.056	-1657.2	17032.69	-0.097
18	20850.0	22507.2	5679.056	-1657.2	17032.69	-0.097
19	34950.0	23967.4	5677.747	10982.6	17033.13	0.645
20	34950.0	23967.4	5677.747	10982.6	17033.13	0.645
21	23000.0	27090.2	4784.187	-4090.2	17305.37	-0.236
22	45000.0	39421.5	4494.104	5578.5	17382.96	0.321
23	115000	108874	3937.760	6126.5	17517.37	0.350
24	130000	129189	4417.071	810.6	17402.69	0.047
25	140000	146316	5208.415	-6316.5	17182.45	-0.368
26	155000	161406	6087.137	-6405.8	16891.15	-0.379
27	175000	177181	6728.449	-2181.4	16646.09	-0.131
28	190000	189726	7580.933	273.7	16275.55	0.017
29	75000.0	43446.4	5570.586	31553.6	17068.47	1.849
30	85000.0	52489.3	5223.965	32510.7	17177.73	1.893
31	95000.0	57045.4	5057.194	37954.6	17227.56	2.203
32	49000.0	61588.2	4913.142	-12588.2	17269.20	-0.729
33	65000.0	69909.9	4729.933	-4909.9	17320.28	-0.283
34	55000.0	51325.4	5170.150	3674.6	17194.00	0.214
35	75000.0	57810.7	4954.235	17189.3	17257.45	0.996
36	80000.0	114004	5368.817	-34003.6	17133.01	-1.985
37	133000	146430	7090.893	-13430.1	16494.95	-0.814
38	172000	163557	8199.177	8442.8	15973.03	0.529
39	155000	192762	6766.995	-37762.0	16630.45	-2.271
40	183000	217534	6212.971	-34533.8	16845.27	-2.050

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	238230	6230.264	-28230.2	16838.88	-1.676
42	43000.0	57142.4	5061.159	-14142.4	17226.40	-0.821
43	105000	101431	4763.779	3569.5	17311.00	0.206
44	173000	128295	5711.267	44705.3	17021.92	2.626
45	300000	284626	10075.24	15374.2	14861.15	1.035
46	330000	325258	9196.816	4742.5	15420.21	0.308
47	390000	359512	9342.047	30488.3	15332.66	1.988
48	62000.0	.	.	.	.	.
49	122000	.	.	.	.	.
50	144000	.	.	.	.	.
51	235000	.	.	.	.	.
52	100000	.	.	.	.	.

<table>
<tr><th>Obs</th><th>-2-1-0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>1</td><td>| | |</td><td>0.004</td><td>-0.4699</td><td>0.0853</td><td>1.2005</td><td>-0.1435</td></tr>
<tr><td>2</td><td>| | |</td><td>0.004</td><td>-0.4633</td><td>0.0821</td><td>1.1973</td><td>-0.1385</td></tr>
<tr><td>3</td><td>| | |</td><td>0.004</td><td>-0.4517</td><td>0.0795</td><td>1.1954</td><td>-0.1327</td></tr>
<tr><td>4</td><td>| *| |</td><td>0.006</td><td>-0.5356</td><td>0.1001</td><td>1.2105</td><td>-0.1786</td></tr>
<tr><td>5</td><td>| *| |</td><td>0.006</td><td>-0.5196</td><td>0.1000</td><td>1.2130</td><td>-0.1732</td></tr>
<tr><td>6</td><td>| | |</td><td>0.003</td><td>-0.3836</td><td>0.1000</td><td>1.2311</td><td>-0.1279</td></tr>
<tr><td>7</td><td>| | |</td><td>0.003</td><td>-0.3913</td><td>0.0872</td><td>1.2130</td><td>-0.1209</td></tr>
<tr><td>8</td><td>| | |</td><td>0.003</td><td>-0.4269</td><td>0.0798</td><td>1.1989</td><td>-0.1257</td></tr>
<tr><td>9</td><td>| *| |</td><td>0.004</td><td>-0.5331</td><td>0.0708</td><td>1.1727</td><td>-0.1471</td></tr>
<tr><td>10</td><td>| | |</td><td>0.003</td><td>-0.4455</td><td>0.0681</td><td>1.1815</td><td>-0.1204</td></tr>
<tr><td>11</td><td>| | |</td><td>0.001</td><td>-0.2231</td><td>0.0629</td><td>1.1964</td><td>-0.0578</td></tr>
<tr><td>12</td><td>| | |</td><td>0.000</td><td>-0.0030</td><td>0.0613</td><td>1.2017</td><td>-0.0008</td></tr>
<tr><td>13</td><td>| |* |</td><td>0.010</td><td>0.8568</td><td>0.0639</td><td>1.1028</td><td>0.2239</td></tr>
<tr><td>14</td><td>| | |</td><td>0.000</td><td>-0.0539</td><td>0.1000</td><td>1.2530</td><td>-0.0180</td></tr>
<tr><td>15</td><td>| | |</td><td>0.001</td><td>0.2034</td><td>0.1001</td><td>1.2473</td><td>0.0678</td></tr>
<tr><td>16</td><td>| | |</td><td>0.001</td><td>0.2034</td><td>0.1001</td><td>1.2473</td><td>0.0678</td></tr>
<tr><td>17</td><td>| | |</td><td>0.000</td><td>-0.0961</td><td>0.1000</td><td>1.2520</td><td>-0.0321</td></tr>
<tr><td>18</td><td>| | |</td><td>0.000</td><td>-0.0961</td><td>0.1000</td><td>1.2520</td><td>-0.0321</td></tr>
<tr><td>19</td><td>| |* |</td><td>0.009</td><td>0.6402</td><td>0.1000</td><td>1.1926</td><td>0.2134</td></tr>
<tr><td>20</td><td>| |* |</td><td>0.009</td><td>0.6402</td><td>0.1000</td><td>1.1926</td><td>0.2134</td></tr>
<tr><td>21</td><td>| | |</td><td>0.001</td><td>-0.2337</td><td>0.0710</td><td>1.2062</td><td>-0.0646</td></tr>
<tr><td>22</td><td>| | |</td><td>0.001</td><td>0.3175</td><td>0.0627</td><td>1.1888</td><td>0.0821</td></tr>
<tr><td>23</td><td>| | |</td><td>0.001</td><td>0.3461</td><td>0.0481</td><td>1.1679</td><td>0.0778</td></tr>
<tr><td>24</td><td>| | |</td><td>0.000</td><td>0.0460</td><td>0.0605</td><td>1.2004</td><td>0.0117</td></tr>
<tr><td>25</td><td>| | |</td><td>0.002</td><td>-0.3638</td><td>0.0842</td><td>1.2120</td><td>-0.1103</td></tr>
</table>

Obs	-2-1-0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
26	| | |	0.004	-0.3753	0.1149	1.2529	-0.1353
27	| | |	0.001	-0.1295	0.1404	1.3097	-0.0523
28	| | |	0.000	0.0166	0.1783	1.3727	0.0077
29	| |*** |	0.073	1.9057	0.0963	0.8166	0.6220
30	| |*** |	0.066	1.9552	0.0847	0.7892	0.5946
31	| |****|	0.084	2.3146	0.0793	0.6631	0.6795
32	| *| |	0.009	-0.7248	0.0749	1.1441	-0.2062
33	| | |	0.001	-0.2804	0.0694	1.2006	-0.0766
34	| | |	0.001	0.2113	0.0829	1.2234	0.0635
35	| |* |	0.016	0.9960	0.0761	1.0835	0.2859
36	| ***| |	0.077	-2.0599	0.0894	0.7571	-0.6455
37	| *| |	0.025	-0.8109	0.1560	1.2343	-0.3486
38	| |* |	0.015	0.5240	0.2085	1.3785	0.2690
39	|****| |	0.171	-2.3953	0.1421	0.6831	-0.9747
40	|****| |	0.114	-2.1352	0.1197	0.7564	-0.7875
41	| ***| |	0.077	-1.7148	0.1204	0.9071	-0.6345
42	| *| |	0.012	-0.8177	0.0795	1.1302	-0.2403
43	| | |	0.001	0.2038	0.0704	1.2073	0.0561
44	| |****|	0.155	2.8384	0.1012	0.5118	0.9524
45	| |** |	0.098	1.0354	0.3149	1.4472	0.7020
46	| | |	0.007	0.3042	0.2624	1.5122	0.1814
47	| |*** |	0.294	2.0642	0.2707	0.9435	1.2577
48		.	.	.	.	.
49		.	.	.	.	.
50		.	.	.	.	.
51		.	.	.	.	.
52		.	.	.	.	.

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	CO2 Dfbetas	ION Dfbetas
1	-0.0570	0.0753	-0.0283	-0.0847	0.0375
2	-0.0530	0.0691	-0.0239	-0.0830	0.0349
3	-0.0489	0.0631	-0.0199	-0.0804	0.0323
4	-0.0029	0.0043	-0.0026	-0.0006	-0.1251
5	-0.0022	0.0031	-0.0017	-0.0005	-0.1218
6	0.0003	-0.0008	0.0013	-0.0001	-0.0912
7	-0.0490	0.0653	-0.0257	-0.0707	0.0323
8	-0.0468	0.0608	-0.0203	-0.0760	0.0309
9	-0.0448	0.0534	-0.0066	-0.0931	0.0296
10	-0.0333	0.0379	-0.0002	-0.0772	0.0220
11	-0.0118	0.0113	0.0057	-0.0379	0.0078
12	-0.0001	0.0001	0.0001	-0.0005	0.0001
13	-0.0222	0.0646	-0.1061	0.1369	0.0145
14	0.0001	-0.0001	0.0000	0.0000	-0.0129
15	-0.0016	0.0025	-0.0019	-0.0002	0.0493
16	-0.0016	0.0025	-0.0019	-0.0002	0.0493
17	-0.0004	0.0007	-0.0006	-0.0000	-0.0225
18	-0.0004	0.0007	-0.0006	-0.0000	-0.0225
19	0.0001	-0.0003	0.0006	-0.0001	0.1518
20	0.0001	-0.0003	0.0006	-0.0001	0.1518
21	-0.0204	0.0249	-0.0047	-0.0407	0.0134
22	0.0174	-0.0173	-0.0066	0.0538	-0.0115
23	-0.0044	0.0076	-0.0072	0.0497	0.0029
24	-0.0033	0.0054	-0.0045	0.0064	0.0021
25	0.0441	-0.0726	0.0605	-0.0490	-0.0290
26	0.0630	-0.1035	0.0859	-0.0495	-0.0414
27	0.0262	-0.0425	0.0343	-0.0166	-0.0172
28	-0.0041	0.0066	-0.0054	0.0021	0.0027
29	0.6208	-0.3515	0.2252	-0.3274	-0.4370
30	0.5869	-0.2844	0.1714	-0.3396	-0.4152
31	0.6624	-0.2860	0.1616	-0.4042	-0.4701
32	-0.1974	0.0738	-0.0378	0.1273	0.1406
33	-0.0704	0.0193	-0.0074	0.0499	0.0504
34	0.0624	-0.0287	0.0161	-0.0365	-0.0442
35	0.2745	-0.1044	0.0505	-0.1737	-0.1953
36	-0.2366	-0.3306	0.3437	0.3989	0.1864
37	-0.0175	-0.2648	0.2468	0.1718	0.0241
38	-0.0163	0.2224	-0.2021	-0.1177	0.0023
39	-0.6295	0.4022	-0.6782	0.5592	0.4500
40	-0.3978	0.0986	-0.3875	0.5093	0.2933

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	CO2 Dfbetas	ION Dfbetas
41	-0.2149	-0.0922	-0.1708	0.4207	0.1667
42	-0.2343	0.1017	-0.0579	0.1428	0.1663
43	0.0330	0.0157	-0.0188	-0.0383	-0.0247
44	0.2522	0.5719	-0.5492	-0.5672	-0.2085
45	0.0720	-0.2936	-0.5707	0.0936	-0.0466
46	-0.0195	-0.0179	0.1082	0.0221	0.0130
47	-0.3622	0.2526	0.4307	0.1257	0.2397
48	.	.	.	.	.
49	.	.	.	.	.
50	.	.	.	.	.
51	.	.	.	.	.
52	.	.	.	.	.

Partial Regression Residual Plot

```
      +-------+------+-------+------+-------+------+-------+------+-------+-
 PRICE|                                                                    |
100000+                                                                    +
      |                                                                    |
      |                                                                11 1|
 50000+                                         1                     1    +
      |                                                                1   |
      |               1             1  2     2           1         1 1 1   |
     0+                        11   1 34   1 1 12 221                      +
      |      1   1  1     1           111          11      1       1       |
      |                                                                    |
-50000+                                                                    +
      |                                                                    |
      +-------+------+-------+------+-------+------+-------+------+-------+-
    -0.8   -0.6    -0.4   -0.2     0.0    0.2     0.4    0.6     0.8    1.0

                                INTERCEP
```

```
                         Partial Regression Residual Plot
        -+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
   PRICE|                                                              |
  100000+                                                             1+
        |                                            1              1  |
        |                                    1             1   2       |
        |                                 1          1                 |
        |               1          2 1    1   1                        |
       0+          1  1   1   2 1 222            1                     +
        |              1 1 1 11   2     1                              |
        |    1      11211 2    1                                       |
        |                                                              |
        |            1                                                 |
 -100000+                                                              +
        -+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
        -20   -15   -10   -5     0     5    10    15    20    25    30

                                   SQRTPOW
```

```
                          Partial Regression Residual Plot
        +------+------+------+------+------+------+------+------+------+-
   PRICE|                                                               |
  100000+                                                               +
        |                                                               |
        |                                                            1  |
   50000+                                  1   1    1                   +
        |               1                   1                1          |
        |                             2   1 1                           |
       0+                    1 1   132521232                            +
        |                 1    1      2      1     1      1             |
        |   1   1  1  1                                                 |
   50000+        1        1                                             +
        |                                                               |
        +------+------+------+------+------+------+------+------+------+-
     -2000  -1500  -1000  -500      0     500   1000   1500   2000   2500

                                   WEIGHT
```

```
                    Partial Regression Residual Plot
      +----+----+----+----+----+----+----+----+----+----+----+----+----+---+
 PRICE|                                                                     |
100000+                                                                     +
      |                                                                     |
      |       1                                                             |
 50000+          21                                                         +
      |     1    1                                                          |
      |      1 1 2                        2       1                         |
    0+ 1        2                        6        1 1           1          +
      |  11    1                          2                12111  31        |
      |                                                             61      |
-50000+                                                                     +
      |                                                                     |
      +----+----+----+----+----+----+----+----+----+----+----+----+----+---+
    -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3  0.4  0.5 0.6

                                   CO2
```

```
                    Partial Regression Residual Plot
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---+
 PRICE|                                                                |
 50000+    1                1                                          +
      | 1 1                                                            |
      |     1                                     1                    |
      |                          1                                     |
      |   1          1           1     21                              |
    0+     1 1           122 1 11         11                       2   +
      |    1                   11 1           1   1  1  1          23  |
      |                  1                                         3   |
      |       1     1    1                                             |
      |                                                                |
-50000+                                                                +
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---+
    -0.5  -0.4  -0.3  -0.2  -0.1   0.0   0.1   0.2   0.3   0.4   0.5 0.6

                                   ION
```

```
       +------+------+------+------+------+------+------+------+------+-
  PRICE|                                                               |
 400000+                                                            1  +
       |                                                               |
       |                                             1         1       |
       |                                                               |
       |                                                               |
 200000+                                1    1    1                    +
       |                       1  1     2   1  1                       |
       |                 1     1    2                                  |
       |  1 12  1         1 1                                          |
       | 25 11111 21                                                   |
      0+ 3  32 1                                                       +
       +------+------+------+------+------+------+------+------+------+-
       0     10     20     30     40     50     60     70     80     90
```

SQRTPOW

```
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+---+
 PRICE|                                                               |
400000+                                                              1+
      |                                                               |
      |                                                              2|
      |                                                               |
      |                                                               |
200000+               1                  2                            +
      |1 1          2 1                  1                            |
      |11           2                                                 |
      |241                                                            |
      |2*                                                             |
     0+81                                                             +
      +----+----+-----+-----+-----+-----+-----+-----+-----+-----+-----+
      0   500 1000  1500  2000  2500  3000  3500  4000  4500  5000 5500
```

WEIGHT

```
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
  RESIDUAL|                                                           |
     50000+                                                           +
R         |        1              1                                   |
e         |     1 1                                                  1|
s         |         1                                1                |
i         |     2      1      1   1       1                           |
d       0+     231  1111   1         1         1 1          1         +
u         |     3 24111               1   1  1                        |
a         |                                                           |
l         |                    1          1   1                       |
          |                          1                                |
    -50000+                                                           +
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
             0     10    20    30    40    50    60    70    80    90

                                SQRTPOW
```

```
          --+----+----+----+----+----+----+----+----+----+----+----+---
  RESIDUAL|                                                           |
     50000+                                                           +
R         |    2                                                      |
e         |    2                                                   1  |
s         |   1                                                    1  |
i         |   131        1                                            |
d       0+   1*         12                                         1  +
u         |   *2         2                                            |
a         |                                                           |
l         |   1                           2                           |
          |                               1                           |
    -50000+                                                           +
          --+----+----+----+----+----+----+----+----+----+----+----+---
            0   500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500

                                WEIGHT
```

Appendix F. SAS Output for Model 5

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	5	357183158596	71436631719	279.543	0.0001
Error	41	10477451284	255547592.3		
C Total	46	367660609881			

Root MSE	15985.85601	R-square	0.9715
Dep Mean	89829.36170	Adj R-sq	0.9680
C.V.	17.79580		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	41218	5463.9699817	7.544	0.0001
SQRTPOW	1	2815.232028	207.35349294	13.577	0.0001
WEIGHT	1	-2.144199	6.88766841	-0.311	0.7571
SRPWRWT	1	0.311781	0.09007288	3.461	0.0013
CO2	1	-41833	5502.1001982	-7.603	0.0001
ION	1	-23660	7230.2321664	-3.272	0.0022

Variable	DF	Standardized Estimate	Variance Inflation
INTERCEP	1	0.00000000	0.00000000
SQRTPOW	1	0.71345389	3.97284347
WEIGHT	1	-0.03501396	18.20002980
SRPWRWT	1	0.37298485	16.70499379
CO2	1	-0.23514680	1.37619502
ION	1	-0.10948231	1.61041037

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	14797.3	4691.883	-9797.3	15281.81	-0.641
2	7600.0	17186.8	4601.501	-9586.8	15309.27	-0.626
3	10000.0	19293.7	4525.611	-9293.7	15331.88	-0.606
4	11300.0	21072.9	5059.537	-9772.9	15164.06	-0.644
5	12000.0	21494.4	5058.837	-9494.4	15164.29	-0.626
6	16000.0	23148.4	5057.916	-7148.4	15164.60	-0.471
7	5000.0	13451.1	4744.762	-8451.1	15265.48	-0.554
8	10780.0	19294.8	4525.728	-8514.8	15331.84	-0.555
9	17000.0	27567.0	4268.207	-10567.0	15405.52	-0.686
10	22000.0	30909.0	4182.498	-8909.0	15429.01	-0.577
11	35000.0	39357.4	4010.072	-4357.4	15474.72	-0.282
12	43000.0	43224.5	3956.980	-224.5	15488.38	-0.014
13	85000.0	69168.2	4050.392	15831.8	15464.21	1.024
14	24500.0	24904.9	5057.783	-404.9	15164.64	-0.027
15	30100.0	26390.0	5058.999	3710.0	15164.24	0.245
16	30100.0	26390.0	5058.999	3710.0	15164.24	0.245
17	20850.0	22336.3	5056.609	-1486.3	15165.04	-0.098
18	20850.0	22336.3	5056.609	-1486.3	15165.04	-0.098
19	34950.0	23763.4	5055.546	11186.6	15165.39	0.738
20	34950.0	23763.4	5055.546	11186.6	15165.39	0.738
21	23000.0	27606.3	4262.228	-4606.3	15407.17	-0.299
22	45000.0	39415.6	4001.342	5584.4	15476.98	0.361
23	115000	96897.6	4925.674	18102.4	15208.07	1.190
24	130000	119343	4853.679	10656.9	15231.20	0.700
25	140000	138265	5187.943	1734.5	15120.61	0.115
26	155000	154936	5732.911	63.5814	14922.51	0.004
27	175000	171651	6200.118	3349.3	14734.52	0.227
28	190000	185661	6851.123	4338.9	14443.33	0.300
29	75000.0	48038.1	5134.119	26961.9	15138.97	1.781
30	85000.0	56605.0	4800.750	28395.0	15247.96	1.862
31	95000.0	61129.3	4654.700	33870.7	15293.18	2.215
32	49000.0	65640.4	4528.374	-16640.4	15331.06	-1.085
33	65000.0	73457.8	4334.249	-8457.8	15387.07	-0.550
34	55000.0	56631.2	4851.766	-1631.2	15231.81	-0.107
35	75000.0	63028.9	4661.515	11971.1	15291.10	0.783
36	80000.0	118429	4948.164	-38429.0	15200.77	-2.528
37	133000	150426	6418.093	-17426.4	14640.89	-1.190
38	172000	167327	7380.948	4673.2	14179.89	0.330
39	155000	168195	9309.862	-13195.0	12995.16	-1.015
40	183000	200157	7470.154	-17156.5	14133.10	-1.214

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	226860	6446.794	-16859.8	14628.28	-1.153
42	43000.0	61129.6	4651.118	-18129.6	15294.27	-1.185
43	105000	104700	4345.323	300.4	15383.94	0.020
44	173000	131247	5156.057	41753.5	15131.51	2.759
45	300000	274097	9472.242	25903.5	12877.28	2.012
46	330000	338487	9036.438	-8486.8	13186.75	-0.644
47	390000	392770	12708.47	-2770.4	9697.550	-0.286
48	62000.0	.	.	.	.	.
49	122000	.	.	.	.	.
50	144000	.	.	.	.	.
51	235000	.	.	.	.	.
52	100000	.	.	.	.	.

Obs	-2-1-0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
1	| *| |	0.006	-0.6364	0.0861	1.1946	-0.1954
2	| *| |	0.006	-0.6215	0.0829	1.1936	-0.1868
3	| *| |	0.005	-0.6014	0.0801	1.1944	-0.1775
4	| *| |	0.008	-0.6398	0.1002	1.2124	-0.2135
5	| *| |	0.007	-0.6214	0.1001	1.2166	-0.2073
6	| | |	0.004	-0.4669	0.1001	1.2474	-0.1557
7	| *| |	0.005	-0.5489	0.0881	1.2157	-0.1706
8	| *| |	0.004	-0.5506	0.0802	1.2049	-0.1625
9	| *| |	0.006	-0.6814	0.0713	1.1652	-0.1888
10	| *| |	0.004	-0.5727	0.0685	1.1854	-0.1552
11	| | |	0.001	-0.2784	0.0629	1.2233	-0.0721
12	| | |	0.000	-0.0143	0.0613	1.2353	-0.0037
13	| |** |	0.012	1.0244	0.0642	1.0609	0.2683
14	| | |	0.000	-0.0264	0.1001	1.2886	-0.0088
15	| | |	0.001	0.2418	0.1002	1.2775	0.0807
16	| | |	0.001	0.2418	0.1002	1.2775	0.0807
17	| | |	0.000	-0.0968	0.1001	1.2868	-0.0323
18	| | |	0.000	-0.0968	0.1001	1.2868	-0.0323
19	| |* |	0.010	0.7335	0.1000	1.1893	0.2445
20	| |* |	0.010	0.7335	0.1000	1.1893	0.2445
21	| | |	0.001	-0.2956	0.0711	1.2322	-0.0818
22	| | |	0.001	0.3570	0.0627	1.2138	0.0923
23	| |** |	0.025	1.1966	0.0949	1.0376	0.3876
24	| |* |	0.008	0.6953	0.0922	1.1886	0.2216
25	| | |	0.000	0.1133	0.1053	1.2937	0.0389

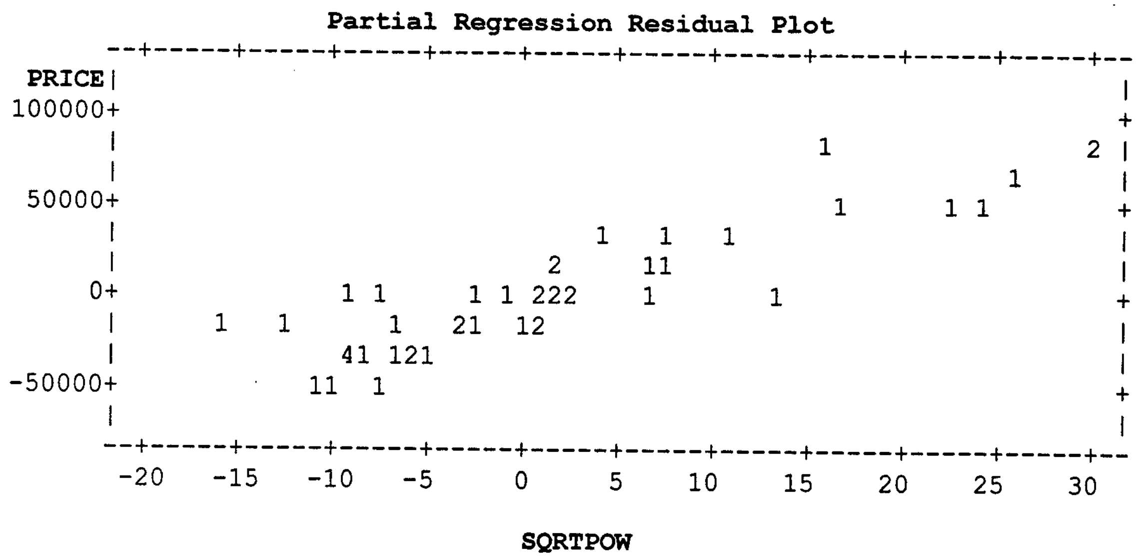
Obs	-2-1-0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
26	| | |	0.000	0.0042	0.1286	1.3309	0.0016
27	| | |	0.002	0.2247	0.1504	1.3547	0.0945
28	| | |	0.003	0.2971	0.1837	1.4020	0.1409
29	| |*** |	0.061	1.8314	0.1031	0.7976	0.6211
30	| |*** |	0.057	1.9225	0.0902	0.7501	0.6053
31	| |**** |	0.076	2.3315	0.0848	0.5899	0.7096
32	| **| |	0.017	-1.0878	0.0802	1.0585	-0.3213
33	| *| |	0.004	-0.5449	0.0735	1.1974	-0.1535
34	| | |	0.000	-0.1058	0.0921	1.2752	-0.0337
35	| |* |	0.009	0.7791	0.0850	1.1580	0.2375
36	|*****| |	0.113	-2.7179	0.0958	0.4640	-0.8847
37	| **| |	0.045	-1.1965	0.1612	1.1196	-0.5245
38	| | |	0.005	0.3260	0.2132	1.4506	0.1697
39	| **| |	0.088	-1.0158	0.3392	1.5062	-0.7277
40	| **| |	0.069	-1.2212	0.2184	1.1911	-0.6455
41	| **| |	0.043	-1.1573	0.1626	1.1366	-0.5100
42	| **| |	0.022	-1.1914	0.0847	1.0277	-0.3623
43	| | |	0.000	0.0193	0.0739	1.2521	0.0054
44	| |**** |	0.147	3.0204	0.1040	0.3773	1.0292
45	| |**** |	0.365	2.0928	0.3511	0.9581	1.5394
46	| *| |	0.032	-0.6389	0.3195	1.6035	-0.4378
47	| | |	0.023	-0.2825	0.6320	3.1139	-0.3702
48		.	.	.	.	.
49		.	.	.	.	.
50		.	.	.	.	.
51		.	.	.	.	.
52		.	.	.	.	.

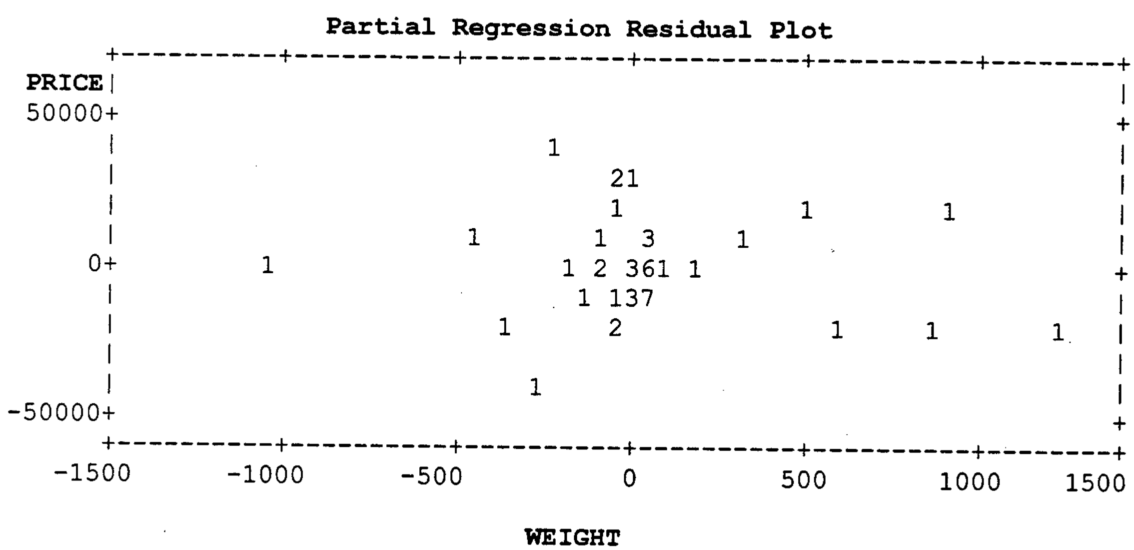
Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	SRPWRWT Dfbetas	CO2 Dfbetas	ION Dfbetas
1	-0.0795	0.1031	0.0015	-0.0199	-0.1089	0.0537
2	-0.0732	0.0937	0.0026	-0.0181	-0.1059	0.0494
3	-0.0670	0.0848	0.0034	-0.0163	-0.1022	0.0452
4	-0.0055	0.0056	0.0050	-0.0070	0.0007	-0.1456
5	-0.0046	0.0041	0.0052	-0.0067	0.0007	-0.1419
6	-0.0012	-0.0006	0.0050	-0.0049	0.0007	-0.1081
7	-0.0708	0.0926	0.0002	-0.0173	-0.0942	0.0478
8	-0.0609	0.0790	-0.0010	-0.0113	-0.0941	0.0412
9	-0.0594	0.0692	0.0110	-0.0161	-0.1138	0.0401
10	-0.0444	0.0494	0.0107	-0.0119	-0.0952	0.0300
11	-0.0147	0.0142	0.0051	-0.0023	-0.0460	0.0100
12	-0.0006	0.0005	0.0003	-0.0000	-0.0024	0.0004
13	-0.0305	0.0781	-0.0388	-0.0172	0.1640	0.0202
14	0.0001	-0.0001	-0.0002	0.0003	-0.0000	-0.0062
15	-0.0021	0.0030	-0.0001	-0.0009	-0.0001	0.0577
16	-0.0021	0.0030	-0.0001	-0.0009	-0.0001	0.0577
17	-0.0003	0.0007	-0.0006	0.0003	-0.0001	-0.0224
18	-0.0003	0.0007	-0.0006	0.0003	-0.0001	-0.0224
19	-0.0008	-0.0002	0.0029	-0.0029	0.0004	0.1714
20	-0.0008	-0.0002	0.0029	-0.0029	0.0004	0.1714
21	-0.0254	0.0316	0.0000	-0.0029	-0.0500	0.0172
22	0.0186	-0.0195	-0.0031	-0.0000	0.0594	-0.0127
23	-0.0988	0.0434	0.2351	-0.2722	0.2246	0.0606
24	-0.0877	0.0905	0.0875	-0.1298	0.1205	0.0565
25	-0.0186	0.0239	0.0076	-0.0174	0.0184	0.0122
26	-0.0008	0.0012	0.0001	-0.0005	0.0006	0.0006
27	-0.0510	0.0756	-0.0036	-0.0244	0.0331	0.0341
28	-0.0770	0.1201	-0.0193	-0.0242	0.0418	0.0517
29	0.6192	-0.3482	-0.0520	0.1605	-0.3405	-0.4440
30	0.5968	-0.2891	-0.0630	0.1499	-0.3572	-0.4302
31	0.6920	-0.2993	-0.0926	0.1799	-0.4349	-0.5001
32	-0.3083	0.1159	0.0507	-0.0831	0.2039	0.2234
33	-0.1416	0.0396	0.0266	-0.0363	0.1022	0.1033
34	-0.0332	0.0151	0.0062	-0.0106	0.0201	0.0238
35	0.2289	-0.0865	-0.0524	0.0768	-0.1486	-0.1651
36	-0.3685	-0.4232	0.4015	-0.2286	0.5618	0.2850
37	-0.0537	-0.3856	0.2417	-0.0943	0.2676	0.0525
38	-0.0019	0.1370	-0.0766	0.0250	-0.0769	-0.0032
39	-0.1180	0.1605	-0.6417	0.5548	0.1607	0.1104
40	-0.0957	0.0335	-0.4927	0.4338	0.2217	0.0942

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	WEIGHT Dfbetas	SRPWRWT Dfbetas	CO2 Dfbetas	ION Dfbetas
41	-0.0611	-0.0794	-0.2855	0.2599	0.2368	0.0649
42	-0.3534	0.1537	0.0449	-0.0897	0.2219	0.2554
43	0.0033	0.0014	-0.0018	0.0012	-0.0038	-0.0025
44	0.3082	0.5981	-0.4045	0.1702	-0.6258	-0.2500
45	-0.0105	-0.5788	0.9544	-0.4944	0.2843	-0.0031
46	-0.0167	0.0503	0.0660	-0.1852	-0.0126	0.0065
47	-0.0201	-0.0317	0.2173	-0.2799	0.0290	0.0067
48	.	.	.	.	.	.
49	.	.	.	.	.	.
50	.	.	.	.	.	.
51	.	.	.	.	.	.
52	.	.	.	.	.	.

```
                         Partial Regression Residual Plot
      +------+------+------+------+------+------+------+------+------+-
PRICE |                                                               |
100000+                                                               +
      |                                                               |
      |                                                          11  1|
 50000+                                       1                  1    +
      |                             1                              1  |
      |                     1    1  2    1           1        1 11    |
     0+                 1          43 1 21  211221                    +
      |     1  1 1  1                2  11 1     1                    |
      |                                                               |
-50000+                                                               +
      |                                                               |
      +------+------+------+------+------+------+------+------+------+-
    -0.8   -0.6   -0.4   -0.2    0.0    0.2    0.4    0.6    0.8    1.0

                                  INTERCEP
```


Partial Regression Residual Plot
PRICE
100000
50000
0
-50000
-20
-15
-10
-5
0
5
10
15
20
25
30
SQRTPOW


Partial Regression Residual Plot
PRICE
50000
0
-50000
-1500
-1000
-500
0
500
1000
1500
WEIGHT

```
                    Partial Regression Residual Plot
      +------------+------------+------------+------------+-----------+
 PRICE|                                                               |
 50000+                                                               +
      |                            11                                 |
      |                             11                       1        |
      |                 1            1                                |
      |                1        13  1                                 |
     0+                  1  1 1  8  21      1                         +
      |                   1 1    37 3                                 |
      |                                                               |
      |            1    1            1                                |
      |      1                                                        |
-50000+                                                               +
      +------------+------------+------------+------------+-----------+
  -100000      -50000           0          50000      100000    150000

                                SRPWRWT
```

```
                    Partial Regression Residual Plot
      --+----+----+----+----+----+----+----+----+----+----+----+----+--
 PRICE|                                                               |
100000+                                                               +
      |                                                               |
      |     1                                                         |
 50000+       3                                                       +
      |   1   1                                                       |
      |      12                         2               1             |
     0+   1  12  1   1         1        6                      1  1   +
      |    1                            2 1               1 11132     |
      |                                                        25     |
-50000+                                                               +
      |                                                               |
      --+----+----+----+----+----+----+----+----+----+----+----+----+--
      -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3  0.4  0.5  0.6

                                  CO2
```

Partial Regression Residual Plot

```
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+----+----+
 PRICE|                                                                |
 50000+                          1                                     +
      |      1 11                                                      |
      |                                    1                           |
      |         1                                1                     |
      |        1          1           1   1   1      1                 |
     0+            1            11  1 11 1               1    1 1   2  +
      |         11             112112     1                1        23 |
      |                               1                             3  |
      |                       1                                        |
      |                                                                |
-50000+                                                                +
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+----+----+
   -0.6  -0.5  -0.4  -0.3  -0.2  -0.1   0.0   0.1   0.2   0.3   0.4 0.5

                                 ION
```

```
      +------+------+------+------+------+------+------+------+------+-
 PRICE|                                                                |
400000+                                                            1   +
      |                                                                |
      |                                             1         1        |
      |                                                                |
      |                                                                |
200000+                                1    1    1                     +
      |                       1  1     2   1  1                        |
      |                 1     1    2                                   |
      |   1 12  1        1 1                                           |
      |  25 11111 21                                                   |
     0+  3  32 1                                                       +
      +------+------+------+------+------+------+------+------+------+-
      0     10     20     30     40     50     60     70     80     90

                               SQRTPOW
```

```
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+----+
 PRICE|                                                                 |
400000+                                                                1+
      |                                                                 |
      |                                                                2|
      |                                                                 |
      |                                                                 |
200000+               1                  2                              +
      |1 1          2 1                  1                              |
      |11           2                                                   |
      |241                                                              |
      |2*                                                               |
     0+81                                                               +
      +-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+----+
      0    500  1000  1500  2000  2500  3000  3500  4000  4500      5500

                                WEIGHT
```

```
      ----+----------+----------+----------+----------+----------+-----
 PRICE|                                                                |
400000+                                                          1    +
      |                                                                |
      |                                            1        1          |
      |                                                                |
      |                                                                |
200000+              1     1 1                                         +
      |     2   111    1                                               |
      |     2  11                                                      |
      |     7                                                          |
      |     *                                                          |
     0+     9                                                          +
      ----+----------+----------+----------+----------+----------+-----
          0       100000     200000     300000     400000     500000

                                SRPWRWT
```

```
        --+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
RESIDUAL|                                                           |
   50000+                                                           +
R       |                       1                                   |
e       |     1 11                                 1                |
s       |                   1   1                                   |
i       |     2   1  1              1                               |
d      0+     231  1 11   1             2  1  1 1                  1+
u       |     3 23 12              1                       1        |
a       |        11                1   1   1                        |
l       |                                                           |
        |                    1                                      |
  -50000+                                                           +
        --+-----+-----+-----+-----+-----+-----+-----+-----+-----+----
          0    10    20    30    40    50    60    70    80    90

                                   SQRTPOW
```

```
        -+----+----+----+----+----+----+----+----+----+----+----+----
RESIDUAL|                                                           |
   50000+                                                           +
R       |    1                                                      |
e       |    3                                                  1   |
s       |     1        1                                            |
i       |   13         1                                            |
d      0+   29         22                                       1   +
u       |   *1                          1                       1   |
a       |   12                          2                           |
l       |                                                           |
        |   1                                                       |
  -50000+                                                           +
        -+----+----+----+----+----+----+----+----+----+----+----+----
         0   500 1000 1500 2000 2500 3000 3500 4000 4500 5000 5500

                                  **WEIGHT**
```

```
          ----+---------+---------+---------+---------+---------+------
  RESIDUAL|                                                           |
     50000+                                                           +
R         |      1                                                    |
e         |      3                                  1                 |
s         |      1  1                                                 |
i         |      4   1                                                |
d        0+      *    112                                         1   +
u         |      *         1                                1         |
a         |      3            1 1                                     |
l         |                                                           |
          |      1                                                    |
    -50000+                                                           +
          ----+---------+---------+---------+---------+---------+------
              0      100000    200000    300000    400000    500000

                                 SRPWRWT
```

Appendix G. SAS Output for Model 6

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	4	357158392506	89289598126	357.083	0.0001
Error	42	10502217375	250052794.65		
C Total	46	367660609881			

Root MSE	15813.05773	R-square	0.9714
Dep Mean	89829.36170	Adj R-sq	0.9687
C.V.	17.60344		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEP	1	41017	5366.8732448	7.643	0.0001
SQRTPOW	1	2796.129692	195.92535853	14.271	0.0001
SRPWRWT	1	0.286443	0.03816337	7.506	0.0001
CO2	1	-41574	5380.2575701	-7.727	0.0001
ION	1	-23527	7139.5206489	-3.295	0.0020

Variable	DF	Standardized Estimate	Variance Inflation
INTERCEP	1	0.00000000	0.00000000
SQRTPOW	1	0.70861286	3.62493417
SRPWRWT	1	0.34267271	3.06472687
CO2	1	-0.23369338	1.34483563
ION	1	-0.10886540	1.60476063

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
1	5000.0	14785.9	4641.024	-9785.9	15116.67	-0.647
2	7600.0	17166.7	4551.316	-9566.7	15143.92	-0.632
3	10000.0	19266.9	4475.878	-9266.9	15166.39	-0.611
4	11300.0	21036.4	5003.501	-9736.4	15000.59	-0.649
5	12000.0	21455.0	5002.581	-9455.0	15000.90	-0.630
6	16000.0	23097.3	5000.612	-7097.3	15001.56	-0.473
7	5000.0	13449.0	4693.469	-8449.0	15100.47	-0.560
8	10780.0	19303.3	4476.726	-8523.3	15166.14	-0.562
9	17000.0	27489.8	4214.943	-10489.8	15240.97	-0.688
10	22000.0	30819.6	4127.511	-8819.6	15264.88	-0.578
11	35000.0	39269.4	3956.854	-4269.4	15310.00	-0.279
12	43000.0	43132.8	3903.354	-132.8	15323.73	-0.009
13	85000.0	68986.0	3964.536	16014.0	15308.01	1.046
14	24500.0	24946.2	5001.391	-446.2	15001.30	-0.030
15	30100.0	26387.3	5004.307	3712.7	15000.32	0.248
16	30100.0	26387.3	5004.307	3712.7	15000.32	0.248
17	20850.0	22363.2	5001.219	-1513.2	15001.35	-0.101
18	20850.0	22363.2	5001.219	-1513.2	15001.35	-0.101
19	34950.0	23782.0	5000.549	11168.0	15001.58	0.744
20	34950.0	23782.0	5000.549	11168.0	15001.58	0.744
21	23000.0	27605.6	4216.155	-4605.6	15240.63	-0.302
22	45000.0	39373.5	3955.826	5626.5	15310.27	0.367
23	115000	97827.9	3873.335	17172.1	15331.34	1.120
24	130000	119940	4411.344	10060.5	15185.28	0.663
25	140000	138580	5033.317	1419.6	14990.61	0.095
26	155000	155003	5666.941	-3.4446	14762.74	-0.000
27	175000	171577	6128.595	3423.2	14577.14	0.235
28	190000	185369	6713.108	4631.2	14317.37	0.323
29	75000.0	47904.4	5060.818	27095.6	14981.35	1.809
30	85000.0	56449.4	4723.028	28550.6	15091.25	1.892
31	95000.0	60940.2	4564.996	34059.8	15139.80	2.250
32	49000.0	65417.9	4423.273	-16417.9	15181.81	-1.081
33	65000.0	73224.1	4222.595	-8224.1	15238.85	-0.540
34	55000.0	56355.2	4718.522	-1355.2	15092.66	-0.090
35	75000.0	62708.7	4497.487	12291.3	15159.99	0.811
36	80000.0	117730	4361.641	-37729.9	15199.63	-2.482
37	133000	149506	5634.318	-16505.6	14775.22	-1.117
38	172000	166289	6513.833	5711.1	14409.12	0.396
39	155000	170751	4344.065	-15750.5	15204.67	-1.036
40	183000	201932	4773.446	-18931.7	15075.38	-1.256

Obs	Dep Var PRICE	Predict Value	Std Err Predict	Residual	Std Err Residual	Student Residual
41	210000	227983	5284.821	-17983.1	14903.81	-1.207
42	43000.0	60950.3	4565.427	-17950.3	15139.67	-1.186
43	105000	104245	4047.863	755.4	15286.19	0.049
44	173000	130616	4689.794	42384.4	15101.61	2.807
45	300000	275925	7351.785	24075.3	14000.14	1.720
46	330000	338063	8836.540	-8062.6	13113.67	-0.615
47	390000	390447	10175.67	-447.3	12104.07	-0.037
48	62000.0	.	.	.	.	.
49	122000	.	.	.	.	.
50	144000	.	.	.	.	.
51	235000	.	.	.	.	.
52	100000	.	.	.	.	.

Obs	-2-1-0 1 2	Cook's D	Rstudent	Hat Diag H	Cov Ratio	Dffits
1	| *| |	0.008	-0.6428	0.0861	1.1740	-0.1974
2	| *| |	0.007	-0.6271	0.0828	1.1726	-0.1885
3	| *| |	0.007	-0.6064	0.0801	1.1728	-0.1790
4	| *| |	0.009	-0.6445	0.1001	1.1919	-0.2150
5	| *| |	0.009	-0.6257	0.1001	1.1953	-0.2087
6	| | |	0.005	-0.4687	0.1000	1.2203	-0.1562
7	| *| |	0.006	-0.5549	0.0881	1.1916	-0.1725
8	| *| |	0.006	-0.5574	0.0801	1.1809	-0.1645
9	| *| |	0.007	-0.6839	0.0710	1.1474	-0.1891
10	| *| |	0.005	-0.5731	0.0681	1.1632	-0.1550
11	| | |	0.001	-0.2758	0.0626	1.1923	-0.0713
12	| | |	0.000	-0.0086	0.0609	1.2012	-0.0022
13	| |** |	0.015	1.0473	0.0629	1.0548	0.2712
14	| | |	0.000	-0.0294	0.1000	1.2533	-0.0098
15	| | |	0.001	0.2447	0.1002	1.2445	0.0816
16	| | |	0.001	0.2447	0.1002	1.2445	0.0816
17	| | |	0.000	-0.0997	0.1000	1.2519	-0.0332
18	| | |	0.000	-0.0997	0.1000	1.2519	-0.0332
19	| |* |	0.012	0.7404	0.1000	1.1728	0.2468
20	| |* |	0.012	0.7404	0.1000	1.1728	0.2468
21	| | |	0.001	-0.2989	0.0711	1.2012	-0.0827
22	| | |	0.002	0.3637	0.0626	1.1841	0.0940
23	| |** |	0.016	1.1236	0.0600	1.0312	0.2839
24	| |* |	0.007	0.6580	0.0778	1.1606	0.1912
25	| | |	0.000	0.0936	0.1013	1.2539	0.0314

<table>
<tr><th>Obs</th><th>-2-1-0 1 2</th><th>Cook's D</th><th>Rstudent</th><th>Hat Diag H</th><th>Cov Ratio</th><th>Dffits</th></tr>
<tr><td>26</td><td>| | |</td><td>0.000</td><td>-0.0002</td><td>0.1284</td><td>1.2943</td><td>-0.0001</td></tr>
<tr><td>27</td><td>| | |</td><td>0.002</td><td>0.2322</td><td>0.1502</td><td>1.3187</td><td>0.0976</td></tr>
<tr><td>28</td><td>| | |</td><td>0.005</td><td>0.3200</td><td>0.1802</td><td>1.3590</td><td>0.1500</td></tr>
<tr><td>29</td><td>| |*** |</td><td>0.075</td><td>1.8609</td><td>0.1024</td><td>0.8379</td><td>0.6286</td></tr>
<tr><td>30</td><td>| |*** |</td><td>0.070</td><td>1.9543</td><td>0.0892</td><td>0.7934</td><td>0.6116</td></tr>
<tr><td>31</td><td>| |****|</td><td>0.092</td><td>2.3701</td><td>0.0833</td><td>0.6476</td><td>0.7146</td></tr>
<tr><td>32</td><td>| **| |</td><td>0.020</td><td>-1.0837</td><td>0.0782</td><td>1.0627</td><td>-0.3157</td></tr>
<tr><td>33</td><td>| *| |</td><td>0.004</td><td>-0.5351</td><td>0.0713</td><td>1.1731</td><td>-0.1483</td></tr>
<tr><td>34</td><td>| | |</td><td>0.000</td><td>-0.0887</td><td>0.0890</td><td>1.2371</td><td>-0.0277</td></tr>
<tr><td>35</td><td>| |* |</td><td>0.012</td><td>0.8074</td><td>0.0809</td><td>1.1342</td><td>0.2395</td></tr>
<tr><td>36</td><td>| ****| |</td><td>0.101</td><td>-2.6550</td><td>0.0761</td><td>0.5523</td><td>-0.7619</td></tr>
<tr><td>37</td><td>| **| |</td><td>0.036</td><td>-1.1205</td><td>0.1270</td><td>1.1112</td><td>-0.4273</td></tr>
<tr><td>38</td><td>| | |</td><td>0.006</td><td>0.3923</td><td>0.1697</td><td>1.3334</td><td>0.1774</td></tr>
<tr><td>39</td><td>| **| |</td><td>0.018</td><td>-1.0368</td><td>0.0755</td><td>1.0720</td><td>-0.2962</td></tr>
<tr><td>40</td><td>| **| |</td><td>0.032</td><td>-1.2647</td><td>0.0911</td><td>1.0250</td><td>-0.4005</td></tr>
<tr><td>41</td><td>| **| |</td><td>0.037</td><td>-1.2134</td><td>0.1117</td><td>1.0645</td><td>-0.4303</td></tr>
<tr><td>42</td><td>| **| |</td><td>0.026</td><td>-1.1916</td><td>0.0834</td><td>1.0380</td><td>-0.3593</td></tr>
<tr><td>43</td><td>| | |</td><td>0.000</td><td>0.0488</td><td>0.0655</td><td>1.2068</td><td>0.0129</td></tr>
<tr><td>44</td><td>| |****|</td><td>0.152</td><td>3.0765</td><td>0.0880</td><td>0.4378</td><td>0.9554</td></tr>
<tr><td>45</td><td>| |*** |</td><td>0.163</td><td>1.7622</td><td>0.2161</td><td>0.9990</td><td>0.9254</td></tr>
<tr><td>46</td><td>| *| |</td><td>0.034</td><td>-0.6102</td><td>0.3123</td><td>1.5678</td><td>-0.4112</td></tr>
<tr><td>47</td><td>| | |</td><td>0.000</td><td>-0.0365</td><td>0.4141</td><td>1.9250</td><td>-0.0307</td></tr>
<tr><td>48</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>49</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>50</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>51</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>52</td><td></td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
</table>

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	SRPWRWT Dfbetas	CO2 Dfbetas	ION Dfbetas
1	-0.0807	0.1095	-0.0436	-0.1115	0.0542
2	-0.0741	0.0998	-0.0370	-0.1085	0.0498
3	-0.0676	0.0906	-0.0311	-0.1047	0.0455
4	-0.0050	0.0074	-0.0059	-0.0001	-0.1472
5	-0.0040	0.0060	-0.0047	-0.0001	-0.1434
6	-0.0006	0.0009	-0.0007	-0.0000	-0.1090
7	-0.0721	0.0980	-0.0404	-0.0964	0.0484
8	-0.0622	0.0834	-0.0288	-0.0962	0.0419
9	-0.0588	0.0761	-0.0145	-0.1172	0.0397
10	-0.0434	0.0551	-0.0053	-0.0980	0.0294
11	-0.0140	0.0163	0.0054	-0.0468	0.0096
12	-0.0003	0.0004	0.0003	-0.0015	0.0002
13	-0.0361	0.0713	-0.1247	0.1756	0.0230
14	0.0001	-0.0002	0.0001	0.0000	-0.0069
15	-0.0021	0.0032	-0.0025	-0.0000	0.0585
16	-0.0021	0.0032	-0.0025	-0.0000	0.0585
17	-0.0004	0.0006	-0.0004	-0.0000	-0.0230
18	-0.0004	0.0006	-0.0004	-0.0000	-0.0230
19	-0.0005	0.0007	-0.0006	-0.0000	0.1732
20	-0.0005	0.0007	-0.0006	-0.0000	0.1732
21	-0.0258	0.0335	-0.0067	-0.0512	0.0174
22	0.0187	-0.0218	-0.0068	0.0617	-0.0127
23	-0.0658	0.1090	-0.1285	0.1762	0.0431
24	-0.0731	0.1144	-0.1114	0.1020	0.0483
25	-0.0147	0.0226	-0.0203	0.0144	0.0097
26	0.0000	-0.0001	0.0001	-0.0000	-0.0000
27	-0.0535	0.0806	-0.0667	0.0352	0.0355
28	-0.0858	0.1287	-0.1044	0.0486	0.0569
29	0.6271	-0.3866	0.2692	-0.3418	-0.4487
30	0.6031	-0.3273	0.2205	-0.3573	-0.4341
31	0.6967	-0.3474	0.2281	-0.4326	-0.5033
32	-0.3029	0.1364	-0.0865	0.1975	0.2197
33	-0.1367	0.0488	-0.0281	0.0974	0.0999
34	-0.0273	0.0148	-0.0099	0.0162	0.0197
35	0.2319	-0.1105	0.0711	-0.1471	-0.1678
36	-0.3124	-0.3079	0.3028	0.4900	0.2529
37	-0.0232	-0.3018	0.2659	0.2146	0.0351
38	-0.0130	0.1402	-0.1210	-0.0774	0.0015
39	-0.1686	-0.0266	-0.0505	0.2248	0.1283
40	-0.1490	-0.1129	-0.0257	0.2877	0.1187

Obs	INTERCEP Dfbetas	SQRTPOW Dfbetas	SRPWRWT Dfbetas	CO2 Dfbetas	ION Dfbetas
41	-0.0973	-0.1746	0.0046	0.2882	0.0835
42	-0.3503	0.1747	-0.1148	0.2175	0.2531
43	0.0079	0.0023	-0.0028	-0.0090	-0.0061
44	0.2646	0.5056	-0.4604	-0.5767	-0.2286
45	0.0791	-0.2377	0.6583	0.1086	-0.0458
46	-0.0085	0.0695	-0.2784	-0.0216	0.0024
47	0.0006	0.0035	-0.0200	-0.0004	-0.0006
48	.	.	.	.	.
49	.	.	.	.	.
50	.	.	.	.	.
51	.	.	.	.	.
52	.	.	.	.	.

```
                         Partial Regression Residual Plot
      +------+------+------+------+------+------+------+------+------+-
 PRICE|                                                               |
100000+                                                               +
      |                                                               |
      |                                                          11  1|
 50000+                                     1                   1     +
      |                                 1                         1   |
      |                       1  1  2    1          1        1  11    |
     0+                  1       1 341  11  12 231  1                 +
      |    1  1  1   1               21     1   1                     |
      |                                                               |
-50000+                                                               +
      |                                                               |
      +------+------+------+------+------+------+------+------+------+-
   -0.8   -0.6   -0.4   -0.2    0.0    0.2    0.4    0.6    0.8    1.0

                                  INTERCEP
```

Partial Regression Residual Plot

```
        --+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
PRICE |                                                                 |
100000+                                                                 +
      |                                       1              11    1|
      |                                                    1            |
 50000+                                        1     1                  +
      |                           1  1                 1                |
      |                     2   1         1                             |
     0+           11 1    1    213      1                               +
      |       1      1    11 1  2  1       1                            |
      |           12   23                                               |
-50000+          211 1                                                  +
      |                                                                 |
        --+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
         -20   -15   -10    -5     0     5    10    15    20    25    30

                                   SQRTPOW
```

Partial Regression Residual Plot

```
      +--------+--------+--------+--------+--------+--------+--------+-
PRICE |                                                                |
100000+                                                                +
      |                                                                |
      |                                                   1            |
 50000+                                  1  1                   1      +
      |             1                     1                  1         |
      |                         2       1 1                            |
     0+                11    1 14412 4211                              +
      |           1  1                13 1                             |
      |      111 1                                                     |
-50000+          1          1                                          +
      |                                                                |
      +--------+--------+--------+--------+--------+--------+--------+-
  -150000  -100000   -50000      0      50000   100000   150000  200000

                                 SRPWRWT
```

```
               Partial Regression Residual Plot
      --+----+----+----+----+----+----+----+----+----+----+----+----+--
 PRICE|                                                               |
100000+                                                               +
      |                                                               |
      |            1                                                  |
 50000+            3                                                  +
      |            2                                                  |
      |     1 1    3                         2       1                |
     0+      1     3                         61                    2 1+
      |            1                         2   1               13  4|
      |                                                              7|
-50000+                                                               +
      |                                                               |
      --+----+----+----+----+----+----+----+----+----+----+----+----+--
      -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1  0.0  0.1  0.2  0.3  0.4  0.5

                                CO2
```

```
               Partial Regression Residual Plot
      --+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
 PRICE|                                                               |
 50000+                        1                                      +
      |   1 11                                                        |
      |                              1                                |
      |       1                              1                        |
      |     1           1          1    1  1     1                    |
     0+         1            11  1  2     1          1     1 1       2+
      |      11         1   1121 2  1   1               1           23|
      |                                                              3|
      |                     1                                         |
      |                                                               |
-50000+                                                               +
      --+-----+-----+-----+-----+-----+-----+-----+-----+-----+-----+--
      -0.5  -0.4  -0.3  -0.2  -0.1   0.0   0.1   0.2   0.3   0.4   0.5

                                ION
```

```
      +------+------+------+------+------+------+------+------+------+-
 PRICE|                                                               |
400000+                                                            1  +
      |                                                               |
      |                                             1         1       |
      |                                                               |
      |                                                               |
200000+                                1    1    1                    +
      |                       1  1     2   1  1                       |
      |                 1     1    2                                  |
      |   1 12  1        1 1                                          |
      |  25 11111 21                                                  |
     0+  3  32 1                                                      +
      +------+------+------+------+------+------+------+------+------+-
      0     10     20     30     40     50     60     70     80     90

                                 SQRTPOW
```

```
      ----+----------+----------+----------+----------+----------+-----
 PRICE|                                                               |
400000+                                                           1   +
      |                                                               |
      |                                           1        1          |
      |                                                               |
      |                                                               |
200000+             1     1 1                                         +
      |     2    111   1                                              |
      |     2   11                                                    |
      |     7                                                         |
      |     *                                                         |
     0+     9                                                         +
      ----+----------+----------+----------+----------+----------+-----
          0      100000     200000     300000     400000     500000

                                 SRPWRWT
```

```
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
  RESIDUAL|                                                           |
     50000+                                                           +
R         |                        1                                  |
e         |     1 11                                                  |
s         |                    1   1                  1               |
i         |     2    1  1              1   1                          |
d       0+     231   1 11   1             1  1  1 1                 1+
u         |     3 23 12                                       1       |
a         |        11                 11   1   1                      |
l         |                                                           |
          |                     1                                     |
    -50000+                                                           +
          ---+-----+-----+-----+-----+-----+-----+-----+-----+-----+---
             0    10    20    30    40    50    60    70    80    90

                                 SQRTPOW
```

```
          -----+---------+---------+---------+---------+---------+-----
  RESIDUAL|                                                           |
     50000+                                                           +
R         |      1                                                    |
e         |      3                                                    |
s         |      1  1                                1                |
i         |      5   1                                                |
d       0+      *    112                                          1  +
u         |      *                                         1          |
a         |      3         1  1 1                                     |
l         |                                                           |
          |      1                                                    |
    -50000+                                                           +
          -----+---------+---------+---------+---------+---------+-----
               0      100000    200000    300000    400000    500000

                                SRPWRWT
```

Appendix H. Work Breakdown Structure/Cost Element Structure Description

(WBS/CES): Aircraft System

WBS/CES Number	WBS/CES Description
1.0	DEATAC Engineering Manufacuring and Development Phase (EMD)
1.1	Aircraft System
1.1.1	Platform
1.1.1.1	Airframe
1.1.1.2	Propulsion
1.1.1.3	Air Vehicle Applications Software
1.1.1.4	Air Vehicle System Software
1.1.1.5	Communications/Identification
1.1.1.6	Navigation and Guidance
1.1.1.7	Central Computer
1.1.1.8	Fire Control
1.1.1.9	Data Display and Controls
1.1.1.10	Survivability
1.1.1.11	Reconnaissance
1.1.1.12	Automatic Flight Controls
1.1.1.13	Central Integrated Checkout
1.1.1.14	Antisubmarine Warfare
1.1.1.15	Armament
1.1.1.16	Weapons Delivery Equipment
1.1.1.17	Auxiliary Equipment
1.1.2	Platform Modification
1.1.2.1	Power System Modification
1.1.2.2	Airframe Modification
1.1.2.3	Thermal Management
1.1.2.3.1	Coolers
1.1.2.3.2	Shielding
1.1.2.3.3	Heat Containment
1.1.2.3.4	Command and Data Handling
1.1.3	Training
1.1.3.1	Equipment
1.1.3.2	Services
1.1.3.3	Facilities
1.1.4	Peculiar Support Equipment (PSE)
1.1.4.1	Test and Measurement Equipment
1.1.4.2	Support and Handling Equipment
1.1.5	Systems Test and Evaluation
1.1.5.1	Development Test and Evaluation
1.1.5.2	Operational Test and Evaluation

WBS/CES Number				WBS/CES Description
		1.1.5.3		Mock-Ups
		1.1.5.4		Test and Evaluation Support
		1.1.5.5		Test Facilities
	1.1.6			Systems Engineering and Project Management
		1.1.6.1		Systems Engineering
		1.1.6.2		Project Management
	1.1.7			Data
		1.1.7.1		Technical Publications
		1.1.7.2		Engineering Data
		1.1.7.3		Management Data
		1.1.7.4		Support Data
		1.1.7.5		Data Depository
	1.1.8			Operational Site Activation
		1.1.8.1		System Assembly, Installation, and Checkout-on-Site
		1.1.8.2		Contractor Technical Support
		1.1.8.3		Site Construction
		1.1.8.4		Site Conversion
	1.1.9			Common Support Equipment (CSE)
		1.1.9.1		Test and Measurement Equipment
		1.1.9.2		Support and Handling Equipment
	1.1.10			Industrial Facilities
		1.1.10.1		Construction, Conversion, Expansion
		1.1.10.2		Equipment Acquisition or Modernization
		1.1.10.3		Maintenance (Industrial Facilities)
1.2				Payload
	1.2.1			Laser
		1.2.1.1		Pump or Pump Source
		1.2.1.2		Gain Medium
		1.2.1.3		Optical Resonator
		1.2.1.4		Non-Linear Optics
		1.2.1.5		Output Optics
		1.2.1.6		Laser Subsystem
			1.2.1.6.1	Gain Medium
			1.2.1.6.2	Cavity
			1.2.1.6.3	Power I/F
		1.2.1.7		Power Source
		1.2.1.8		Beam Control
			1.2.1.8.1	Optical Elements
		1.2.1.9		Transmitting Optics
		1.2.1.10		Targeting, Tracking, and Control
		1.2.1.11		Power Supply
		1.2.1.12		Pumping Device

WBS/CES Number	WBS/CES Description
1.2.1.13	Lasing Medium
1.2.1.14	Optical Resonant Cavity
1.2.2	Laser Mission Equipment
1.2.2.1	Power System Modification
1.2.2.2	Gain Medium
1.2.2.3	Optics
1.2.2.4	Mounting
1.2.2.5	Cooling
1.2.2.6	Control
1.2.2.7	Computers
1.2.2.8	Software
1.2.2.9	Display
1.2.2.10	Sensors
1.2.2.11	Communications
1.2.2.12	Integration and Assembly
1.2.3	Training
1.2.3.1	Equipment
1.2.3.2	Services
1.2.3.3	Facilities
1.2.4	Peculiar Support Equipment (PSE)
1.2.4.1	Test and Measurement Equipment
1.2.4.2	Support and Handling Equipment
1.2.5	Systems Test and Evaluation
1.2.5.1	Development Test and Evaluation
1.2.5.2	Operational Test and Evaluation
1.2.5.3	Mock-Ups
1.2.5.4	Test and Evaluation Support
1.2.5.5	Test Facilities
1.2.6	Systems Engineering and Project Management
1.2.6.1	Systems Engineering
1.2.6.2	Project Management
1.2.7	Data
1.2.7.1	Technical Publications
1.2.7.2	Engineering Data
1.2.7.3	Management Data
1.2.7.4	Support Data
1.2.7.5	Data Depository
1.2.8	Operational Site Activation
1.2.8.1	Systems Assembly, Installation, and Checkout On-Site
1.2.8.2	Contractor Technical Support
1.2.8.3	Site Construction
1.2.8.4	Site Conversion

WBS/CES Number	WBS/CES Description
1.2.9	Common Support Equipment (CSE)
1.2.9.1	Test and Measurement Equipment
1.2.9.2	Support and Handling Equipment
1.2.10	Industrial Facilities
1.2.10.1	Construction, Conversion, Expansion
1.2.10.2	Equipment Acquisition or Modernization
1.2.10.3	Maintenance (Industrial Facilities)
1.3	Payload and Platform Integration
1.4	Engineering Change Orders
1.5	SPO Support
1.5.1	Government Test Agency Support
1.5.2	System/Project Management Support
1.5.2.1	SW Independent Verification and Validation (IV&V)
1.5.2.2	MITRE, TEMS, etc.
1.5.2.3	Other
1.6	Warranty
2.0	DEATAC Production Phase
2.1	Aircraft System
2.1.1	Air Vehicle
2.1.1.1	Airframe
2.1.1.2	Propulsion
2.1.1.3	Air Vehicle Applications Software
2.1.1.4	Air Vehicle System Software
2.1.1.5	Communications/Identification
2.1.1.6	Navigation/Guidance
2.1.1.7	Central Computer
2.1.1.8	Fire Control
2.1.1.9	Data Display and Control
2.1.1.10	Survivability
2.1.1.11	Reconnaissance
2.1.1.12	Automatic Flight Control
2.1.1.13	Central Integrated Checkout
2.1.1.14	Antisubmarine Warfare
2.1.1.15	Armament
2.1.1.16	Weapons Delivery Equipment
2.1.1.17	Auxiliary Equipment
2.1.2	Platform Modification
2.1.2.1	Power System Modification
2.1.2.2	Aiframe Modification
2.1.2.3	Thermal Management
2.1.2.3.1	Coolers
2.1.2.3.2	Shielding
2.1.2.3.3	Heat Containment

WBS/CES Number	WBS/CES Description
2.1.2.4	Command and Data Handling
2.1.3	Training
2.1.3.1	Equipment
2.1.3.2	Services
2.1.3.3	Facilities
2.1.4	Peculiar Support Equipment (PSE)
2.1.4.1	Test and Measurement Equipment
2.1.4.2	Support and Handling Equipment
2.1.5	Systems Test and Evaluation
2.1.5.1	Development Test and Evaluation
2.1.5.2	Operational Test and Evaluation
2.1.5.3	Mock-Ups
2.1.5.4	Test and Evaluation Support
2.1.5.5	Test Facilities
2.1.6	System Engineering and Project Management
2.1.6.1	Systems Engineering
2.1.6.2	Project Management
2.1.7	Data
2.1.7.1	Technical Publications
2.1.7.2	Engineering Data
2.1.7.3	Management Data
2.1.7.4	Support Data
2.1.7.5	Data Depository
2.1.8	Operational Site Activation
2.1.8.1	System Engineering and Project Management
2.1.8.2	Contractor Technical Support
2.1.8.3	Site Construction
2.1.8.4	Site Conversion
2.1.9	Common Support Equipment (CSE)
2.1.9.1	Test and Measurement Equipment
2.1.9.2	Support and Handling Equipment
2.1.10	Industrial Facilities
2.1.10.1	Construction, Conversion, Expansion
2.1.10.2	Equipment Acquisition or Modernization
2.1.10.3	Maintenance (Industrial Facilities)
2.2	Payload
2.2.1	**Laser Device**
2.2.1.1	**Pump or Pump Source**
2.2.1.2	**Gain Medium**
2.2.1.3	**Optical Resonator**
2.2.1.4	**Non-Linear Optics**
2.2.1.5	**Output Optics**
2.2.1.6	**Laser Subsystem**

WBS/CES Number			WBS/CES Description
		2.2.1.6.1	**Gain Medium**
		2.2.1.6.2	**Cavity**
		2.2.1.6.3	**Power I/F**
	2.2.1.7		**Power Source**
	2.2.1.8		**Beam Control**
		2.2.1.8.1	**Optical Elements**
	2.2.1.9		**Transmitting Optics**
	2.2.1.10		**Targeting, Tracking, and Control**
	2.2.1.11		**Power Supply**
	2.2.1.12		**Pumping Device**
	2.2.1.13		**Lasing Medium**
	2.2.1.14		**Optical Resonant Cavity**
2.2.2			Laser Mission Equipment
	2.2.2.1		Power System
	2.2.2.2		Gain Medium
	2.2.2.3		Optics
	2.2.2.4		Mounting
	2.2.2.5		Cooling
	2.2.2.6		Control
	2.2.2.7		Computers
	2.2.2.8		Software
	2.2.2.9		Display
	2.2.2.10		Sensors
	2.2.2.11		Communications
	2.2.2.12		Integration and Assembly
2.2.3			Training
	2.2.3.1		Equipment
	2.2.3.2		Services
	2.2.3.3		Facilities
2.2.4			Peculiar Support Equipment (PSE)
	2.2.4.1		Test and Measurement Equipment
	2.2.4.2		Support and Handling Equipment
2.2.5			System Test and Evaluation
	2.2.5.1		Development Test and Evaluation
	2.2.5.2		Operational Test and Evaluation
	2.2.5.3		Mock-Ups
	2.2.5.4		Test and Evaluation Support
	2.2.5.5		Test Facilities
2.2.6			Systems Engineering and Project Management
	2.2.6.1		Systems Engineering
	2.2.6.2		Project Management

WBS/CES Number	WBS/CES Description
2.2.7	Data
2.2.7.1	Technical Publications
2.2.7.2	Engineering Data
2.2.7.3	Management Data
2.2.7.4	Support Data
2.2.7.5	Data Depository
2.2.8	Operational Site Activation
2.2.8.1	System Assembly, Installation, and Check-Out On Site
2.2.8.2	Contractor Technical Support
2.2.8.3	Site Construction
2.2.8.4	Site Conversion
2.2.9	Common Support Equipment (CSE)
2.2.9.1	Test and Measurement Equipment
2.2.9.2	Support and Handling Equipment
2.2.10	Industrial Facilities
2.2.10.1	Construction, Conversion, Expansion
2.2.10.2	Equipment Acquisition or Modernization
2.2.10.3	Maintenance (Industrial Facilities)
2.3	Payload and Platform Integration
2.4	Engineering Change Orders
2.5	SPO Support
2.5.1	Government Test Agency Support
2.5.2	System and Project Management Support
2.5.2.1	SW Independent Verification and Validation (IV&V)
2.5.2.2	Mitre, TEMS, etc.
2.5.2.3	Other
2.5.3	Travel
2.6	Warranty
3.0	DEATAC Operations and Support Phase
3.1	Mission Personnel
3.1.1	Operations
3.1.1.1	Officers
3.1.1.2	Enlisted
3.1.1.3	Civilians
3.1.2	Maintenance
3.1.2.1	Officers
3.1.2.2	Enlisted
3.1.2.3	Civilians
3.1.3	Other Mission Personnel
3.1.3.1	Unit Staff
3.1.3.1.1	Officers
3.1.3.1.2	Enlisted
3.1.3.1.3	Civilians

WBS/CES Number	WBS/CES Description
3.1.3.2	Security
3.1.3.2.1	Officers
3.1.3.2.2	Enlisted
3.1.3.2.3	Civilians
3.1.3.3	Other Support
3.1.3.3.1	Officers
3.1.3.3.2	Enlisted
3.1.3.3.3	Civilians
3.2	Unit Staff
3.2.1	Petroleum, Oil , Lubricant (POL) and Energy Consumption
3.2.2	POL
3.2.2.1	Propulsion Fuel
3.2.2.2	Field Generated Electricity
3.2.2.3	Commercial Electricity
3.2.3	Consumable Material and Repair Parts
3.2.3.1	Maintenance Material
3.2.3.2	Operational Material
3.2.3.3	Mission Support Supplied
3.2.4	Depot Level Reparables
3.2.4.1	Transportation
3.2.4.2	Inventory Losses
3.2.4.3	Inventory Maintenance
3.2.4.4	Inventory Control Point
3.2.4.5	Condemnations
3.2.4.6	Price Stabilization
3.2.5	Training Munitions and Expendable Stores
3.2.5.1	Ammunition (Live and Inert)
3.2.5.2	Bombs
3.2.5.3	Rockets
3.2.5.4	Training Missiles
3.2.5.5	Sonobuoys
3.2.5.6	Pyrotechnics
3.2.6	Other
3.2.6.1	Purchased Services
3.2.6.1.1	Special Support Equipment
3.2.6.1.2	Communication Circuits
3.2.6.1.3	Vehicles
3.2.6.1.4	Custodial Services
3.2.6.1.5	Computers and Administration Equipment
3.2.6.2	Transportation
3.2.6.2.1	Resupply (Materials and Repair Parts)
3.2.6.2.2	Training Deployments

WBS/CES Number				WBS/CES Description
		3.2.6.3		TAD and TDY
			3.2.6.3.1	Commercial Transportation
			3.2.6.3.2	Rental Vehicles
			3.2.6.3.3	Mileage Allowances
			3.2.6.3.4	Per Diem Allowances
			3.2.6.3.5	Incidental Travel Expenses
3.3				Intermediate Maintenance
	3.3.1			Maintenance
		3.3.1.1		Officers
		3.3.1.2		Enlisted
		3.3.1.3		Civilians
	3.3.2			Consumable Material and Repair Parts
	3.3.3			Other
3.4				Depot Maintenance
	3.4.1			Overhaul and Rework
		3.4.1.1		Aircraft
			3.4.1.1.1	Airframe
			3.4.1.1.2	Engine Rework
			3.4.1.1.3	Component Repair
		3.4.1.2		Payload
		3.4.1.3		Ground Communication and Electronics
			3.4.1.3.1	Transmitters
			3.4.1.3.2	Receivers
			3.4.1.3.3	Antennas
			3.4.1.3.4	Pedestals
			3.4.1.3.5	Operator Consoles
	3.4.2			Other
3.5				Contractor Support
	3.5.1			Interim Contractor Support (ICS)
	3.5.2			Contractor Logistics Support (CLS)
	3.5.3			Other
3.6				Sustaining Support
	3.6.1			Support Equipment Replacement
	3.6.2			Modification Kit Procurement and Installation
	3.6.3			Other Recurring Investment
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	3.6.5			Software Maintenance Support
	3.6.6			Simulator Operations
	3.6.7			Other

WBS/CES Number				WBS/CES Description
3.7				Indirect Support
	3.7.1			Personnel Support
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			3.7.1.1.1	System Specific Training
			3.7.1.1.1.1	Program Management
			3.7.1.1.1.2	Development
			3.7.1.1.1.3	Engineering
			3.7.1.1.1.4	Manufacturing
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			3.7.1.1.1.7	Subcontract
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			3.7.1.1.2.1	Officers
			3.7.1.1.2.2	Enlisted
			3.7.1.1.2.3	Civilians
		3.7.1.2		Permanent Change of Station (PCS)
			3.7.1.2.1	Officers
			3.7.1.2.2	Enlisted
			3.7.1.2.3	Civilians
		3.7.1.3		Medical Support
			3.7.1.3.1	Officers
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	3.7.2			Installation Support
		3.7.2.1		Base Operating Support (BOS)
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			3.7.2.2.2	Enlisted
			3.7.2.2.3	Civilians
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Captain Michael J. Nolette was born on 13 February 1966 in Providence, Rhode Island. He graduated from Bishop Hendricken High School in 1984 and attended the United States Air Force Academy (USAFA). In June 1988, he graduated from USAFA with a Bachelor of Science in Management and a commission in the United States Air Force.

His first assignment was as an undergraduate pilot at Williams AFB, Arizona. His second assignment was to Rome Laboratory located at Griffiss AFB, New York as a cost analysis officer. His next assignment was to Aeronautical Systems Center as a budget officer in the Training Systems Product Group and subsequently the F-22 System Program Office. Prior to attending the Air Force Institute of Technology, he served in the Logistics Career Broadening Program at the Ogden Air Logistics Center located at Hill Air Force Base, Utah. Upon graduation, he will be assigned to Aeronautical Systems Center Cost Staff located at Wright-Patterson AFB, Ohio.

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13. ABSTRACT *(Maximum 200 words)*
The Post Cold War acquisition environment has been marked by significant budget reductions and the greater use of commercial practices. In this austere and rapidly changing acquisition environment, accurate cost estimates are paramount to maximize the use of the DoD's critical resources.

The advancement of laser technology has spawned a large variety of military laser applications such as the Airborne Laser (ABL), Space-Based Laser (SBL) and a host of other smaller programs. Furthermore, the Air Force Laboratory's Directed Energy Applications for Tactical Airborne Combat (DE ATAC) Study [the sponsoring activity of this thesis research] is considering a host of airborne laser system proposals. While the need for laser cost models/data is clearly increasing, such information is not readily available. Commercial pricing data is used in this research to create models that help to fill this void.

Specifically, this thesis built a commercial laser database containing commercial laser parameters and prices gathered from commercial manufacturing catalogs. An evaluation of the data identified some laser parameters as cost drivers. Using the technique of Ordinary Least Squares Regression, a series of six CERs were developed to estimate laser device prices.

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